Noise Traders, Exchange Rate Disconnect Puzzle, and the Tobin Tax

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Abstract

This paper proposes a framework to explain why the nominal and real exchange rates are highly volatile and seem to be disconnected from macroeconomic fundamentals. Two types of foreign exchange traders, rational traders and noise traders with erroneous stochastic beliefs, are introduced into a sticky-price dynamic general-equilibrium framework. The presence of noise traders creates deviations from the uncovered interest parity. As a result, exchange rates can diverge significantly from the fundamental values. Combined with local currency pricing and consumption smoothing behavior in an infinite horizon model, the presence of noise traders can help to explain the “exchange rate disconnect puzzle”. Then we show that the excess exchange rate volatility caused by the presence of noise traders can be reduced by the ‘Tobin tax’ type of policies. However, the effect of the ‘Tobin tax’ on the exchange rate volatility depends on the market structure of foreign exchange market and the interaction of the Tobin tax with other trading costs.

JEL classification numbers: F41, F31, G15

Keywords: Noise traders; microstructure; exchange rate disconnect; macroeconomic fundamentals; volatility.
1 Introduction

A central puzzle in international macroeconomics over the last 20 years is that real exchange rates are volatile and persistent. Furthermore, as Flood and Rose (1995) have elegantly documented, the exchange rate seems to “have a life of its own”, being disconnected from other macroeconomic variables. For example, Mussa (1986), Baxter and Stockman (1989) and Flood and Rose (1995) all find that both nominal and real exchange rates are highly volatile, especially when compared to the macroeconomic fundamentals, such as relative price level, consumption, and output. Exchange rate volatility also varies substantially over time. Obstfeld and Rogoff (2000) state this kind of “exceedingly weak relationship between the exchange rate and virtually any macroeconomic aggregates” as the “exchange rate disconnect puzzle”.

This irregularity casts some doubts on the traditional monetary macroeconomic model of exchange rates, which assumes that purchasing power parity (PPP) holds. With PPP, the “expenditure-switching” effect of exchange rate changes will lead to substitution between domestically-produced goods and internationally-produced goods. It implies that the exchange rate volatility will be transferred to macroeconomic fundamentals. Nevertheless, empirical evidence \(^1\) indicates that nominal exchange rate changes are not fully passed through to goods prices. Motivated by this evidence, Betts and Devereux (1996, 2000) introduce local currency pricing into the baseline Redux model developed by Obstfeld and Rogoff (1995). They assume that firms can charge different prices for the same goods in home and foreign markets and that the prices are sticky in each country in terms of the local currency. This allows the real exchange rate to fluctuate, and delinks the home and foreign price levels.

Although the new open economy macroeconomic models with sticky prices, imperfect competition and local currency pricing can generate volatile exchange rate movements,\(^2\) they typically predict a strong counterfactual relationship between the real exchange rate and relative consumption. A monetary shock simultaneously raises domestic consumption (by more than it raises foreign consumption) and creates a (temporary) depreciation of home currency. Consequently, these models almost generically predict a strong positive correlation between depreciation and relative consumption, which is not observed empirically.\(^3\) For example, Chari, Kehoe and McGrattan (2002) refer to this puzzle as the consumption-real exchange rate anomaly and

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\(^1\)See Engel (1993, 1999) and Parsley and Wei (2001) for details.

\(^2\)A high risk aversion coefficient of household (about 5) is usually required in these models to reproduce the data’s volatility of real exchange rate relative to output. See Chari, Kehoe and McGrattan (2002).

\(^3\)Benigno and Thoenissen (2003) report the correlation between bilateral exchange rate and bilateral relative consumption for seven countries (Canada, France, West Germany, Italy, Japan, U.K. and U.S.) for the periods starting from 1970 until 2002. The cross-correlation varies between \(-0.45\) and \(0.42\).
they show that neither incomplete financial market nor habit persistence can eliminate it.

One explanation for this discrepancy might lie in the fact that the nominal exchange rate is also an asset price, and therefore will be inevitably affected by imperfections in the financial markets. These imperfections may include herd behavior, momentum investing and noise trading. Working together with sticky prices, these are all important reasons to explain why the real exchange rate persistently deviates from the level predicted by the fundamentals-based models.

Since the early nineties, many economists studied the behavior of markets participants in foreign exchange markets through survey conducted in dealers. The evidence collected in these surveys showed that a majority of traders does not rely on information about fundamentals factors, but use various sources of ‘technical’ information such as trend curves. Also, most traders believe that at least over the short and medium run, exchange rates are governed more by speculative behavior or technical trading rather than macroeconomic fundamental and pertinent news. Evans and Lyons (2002) show that most of the short-run exchange rate volatility is related to order flow, which also reflects the heterogeneity in investors’ expectations. Cai, Cheung, Lee and Melvin(2001) also provide evidence in support of the independent role of order flow and its associated information as a determinant of exchange rate dynamics. These pieces of evidence all suggest deviation from rational expectation and the extensive use of non-fundamental trading strategy in foreign exchange markets.

Therefore, our paper intends to propose a new approach to study exchange rates, that combines the macroeconomic model of exchange rates and the microstructure approach of foreign exchange markets. This approach is implemented within a specific model, where noise traders are introduced into the new open economy macroeconomic framework. The combination is helpful for understanding the behavior of exchange rates and their relationship with macroeconomic fundamentals. It also gives more rigorous microeconomic foundations to the “noise trader” approach and enriches the new open economy macroeconomic framework with a more realistic setting of the microstructure of foreign exchange market. In addition, it provides a well-defined framework for policy evaluations, especially for policies designed to control non-fundamental volatilities, such as the ‘Tobin tax’.

We adopt the overlapping-generation noise trader model of De Long et al. (1990). Two types of foreign exchange traders are introduced into the general equilibrium framework. One type is the “rational/informed trader”, which has rational expectations about future investment returns, while the other type cannot forecast the future returns correctly and is called the “noise trader”.

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The results from the model show that when the number of noise traders increases, so does the exchange rate volatility. Nevertheless, the volatilities of macroeconomic fundamentals (except for the net foreign assets) are completely independent of the noise component on the foreign exchange market. Therefore, our model can generate a relative volatility of real exchange rate to output close to the data, even for a low risk aversion coefficient. Moreover, since in this model nominal and real exchange rate fluctuations can be generated by erroneous belief of noise traders, our model does not predict a strong comovement of exchange rates and fundamentals. Therefore, it is possible to explain the “exchange rate disconnect puzzle” by the approach suggested in this paper.

The basic intuition behind our results is as follows. The heterogeneity in beliefs among foreign exchange traders creates the basis for trading volume and deviations from the uncovered interest parity. This breaks the link between an exchange rate shock and the home relative real interest rates (home relative expected future consumption) implied by the interest rate parity condition. Therefore, exchange rates can diverge significantly from the fundamental values. The greater the number of noise traders, the more volatile will be the exchange rates.

However, why is the exchange rate volatility not transferred to macroeconomic fundamentals? Normally, there are two channels through which the exchange rate affects the macroeconomic variables: the expenditure-switching effect and the wealth effect. Under the assumption of local currency pricing, the expenditure-switching effect is eliminated as the relative price of home and foreign goods does not change. Although the wealth effect still exists, it turns out to be quite small quantitatively. This is because the wealth effect of exchange rate change is spread out over current and future periods through intertemporal consumption smoothing, and so tends to be very small.

Another important implication of our model is related to the consumption-real exchange rate anomaly. We show that we can get a much smaller cross-correlation between exchange rates and relative consumption in our model. Moreover, this cross-correlation decreases when more noise traders are present on the foreign exchange market. Intuitively, this is because our model setting separates the foreign exchange market from the rest of the model, insuring that the expectation error creates a source of fluctuation which only affects exchange rates, but not macroeconomic fundamentals. In other words, both expectation error shocks and monetary shocks cause exchange rate fluctuations, but macroeconomic fundamentals can only be affected by monetary shocks. So our model will not predict a strong comovement of the exchange rate and relative consumption.

Many economists have suggested that increasing the trading cost on the foreign exchange market might reduce the exchange rate volatility. To understand the effect of this kind of
policies, the size of the noise component is endogenized by introducing a heterogeneous entry cost for noise traders. Only noise traders having entry costs that are sufficiently low will choose to enter the foreign exchange market. We find that increasing the entry cost will reduce the exchange rate volatility. We also analyze the ‘Tobin tax’ type of policy suggested by Tobin (1978) and Eichengreen, Tobin and Wyplosz (1995). We find that a Tobin tax will decrease the exchange rate volatility, however, the impact of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs. One important policy implication given the model structure is that, Tobin tax and other kinds of entry cost can reduce the excess volatility without affecting the volatilities of key macroeconomic variables, such as consumption and output.

This paper belongs to the new open economy macroeconomics literature. Wang (2008) demonstrates in a two-country DSGE model, that the home bias in consumption is important to duplicate the exchange rate volatility and exchange rate disconnect documented in the data. When home bias is high, the shock to uncovered interest rate parity can substantially drive up exchange rate volatility while leaving the volatility of real macroeconomic variables almost untouched. The paper that is closest, in spirit, to our analysis of exchange rate disconnect puzzle is Devereux and Engel (2002). They stated that the key ingredients for a general equilibrium model to explain the exchange rate disconnect puzzle include: local currency pricing to eliminate the expenditure-switching effect, a special structure of international pricing and product distribution to minimize the wealth effect, incomplete international financial markets, and stochastic deviations from the uncovered interest parity. The major difference between our paper and theirs is that we introduce more microeconomic foundations of noise traders. In our model, both noise traders and rational traders are risk averse and utility maximizing agents. Therefore, our paper provides a framework to analyze the implication of policies like Tobin tax. Also, instead of assuming a specific production and distribution structure to remove wealth effects completely, we show that the wealth effect is quite small quantitatively in a standard infinite horizon model. Finally, we point out the implication of expectation errors in explaining the consumption-real exchange rate anomaly.

The microstructure of the exchange rate market in this paper follows the noise trader literature initiated by De Long et al. (1990), especially the work of Jeanne and Rose (2002). They also use noise traders to generate non-fundamental based exchange rate volatility. Their paper shows that for the same level of fundamental macroeconomic volatility, there exist multiple equilibria under floating regime. When noise traders are present, exchange rate volatility will be high; while when they are absent from the markets, exchange rate volatility is low. Although they focus on the the role of fixed exchange rate regime in reducing non-fundamental exchange
rate volatility, their results can also be used to explain the exchange rate disconnect puzzle. The macroeconomic part of their model, however, is a simple monetary model of exchange rates with PPP. Neither nominal rigidities nor pricing to market is considered. Moreover, intertemporal optimizing agents and profit maximizing firms are not modeled. So most channels through which the exchange rate affects macroeconomic fundamentals, such as the expenditure-switching effects and wealth effects of exchange rate changes, cannot be analyzed. Therefore, to study the exchange rate disconnect puzzle more rigorously, this model generalized the macroeconomic part of their model along the lines of the “New Open Macroeconomics”. Another feature of their model is that it is a partial equilibrium model without explicit welfare specifications for households, so rigorous policy analysis is impossible.5

As to the discussion of the Tobin tax, our study contributes to the literature on effects of the Tobin tax on foreign exchange rate volatility. Since James Tobin propose the ‘Tobin tax’ in 1974, the debate about the Tobin tax concentrates on its feasibility and the “distorting” effects it might have as a tax. Recently, some scholars question the conventional wisdom that an increase in the Tobin tax reduces market volatility. For example, see Davidson (1997, 1998) and Song and Zhang (2005). The major difference of our paper is that we study the effect of the Tobin tax on the volatility of exchange rates and macroeconomic fundamentals in a DSGE model. Also, we show that although a Tobin tax can reduce exchange rate volatility, its effect depends on the market microstructure and the interaction of the Tobin tax with other trading costs.

This paper is organized as follows. In Section 2, we construct a model that embeds noise traders and the Tobin tax into a new open economy macroeconomic framework. Both the exogenous entry and endogenous entry specifications are explored. In Section 3 features of the solution to the model are discussed. Section 4 gives the results of the model. The paper concludes with a brief summary and suggestions for subsequent research.

5Becchetta and van Wincoop (2004) and Evans and Lyons (2005) introduce order flow and information dispersion about future fundamentals into dynamic macroeconomic model to explain the exchange rate disconnect puzzle. There are several important differences in comparison to our approach. First, our paper follows the NOEM framework, while the macroeconomic framework in their papers is either a partial-equilibrium monetary model (Becchetta and van Wincoop, 2004) or a DGE model without nominal rigidities (Evans and Lyons, 2005), so the monetary policy analysis is not possible. Also, a lot of channels through which exchange rates affect macroeconomic fundamentals are not considered. Second, they focus on information dispersion, and our paper focuses on noise traders. Third, both papers emphasize the role of order flow in the information dispersion. Therefore, their settings of foreign exchange market are different from ours.
2 The model

The world economy consists of two countries, denominated by home and foreign. Each country specializes in the production of a composite traded good. Variables in the foreign country are denoted by an asterisk. In addition, a subscript $h$ denotes a variable originating from the home country; a subscript $f$ denotes a variable produced in the foreign country.

This model is analogous to most new open economy macroeconomic models except for the foreign exchange market. Each country is populated by a large number of atomistic households, a continuum of firms that set prices in advance, and a government (a combined fiscal and monetary authority). However, we assume that home and foreign households can only trade nominal bonds denominated in their domestic currency. Although home households cannot access the international bond market, the foreign exchange traders can carry out the international bond trading to maximize their utility. Thus, besides the infinitely lived household, a second type of representative agent is introduced into the model, namely, the foreign exchange trader, who lives in an overlapping-generation demographic structure. Hereafter, a superscript H denotes households and a superscript T stands for traders. In the foreign country, only one type of representative agent is present; the foreign household.\(^6\)

2.1 Households, Firms and Government

The lifetime expected utility of the home representative household is:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_H)^{1-\rho}}{1-\rho} + \frac{1}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\eta}{1+\psi} L_t^{1+\psi} \right] \right\} \quad (2.1)$$

Subject to

$$P_t C^H_t + B_{t+1} + M_t = W_t L_t + \Pi_t + M_{t-1} + T_t + B_t (1 + r_t) \quad (2.2)$$

where $C^H_t$ is the time $t$ composite consumption of home households, composed by a continuum of home goods and foreign goods; both are of measure 1. Let $C^T_t$ denote the composite consumption of traders, then $C^T_t + C^H_t = C_t$, where $C_t$ is the composite consumption of the home country and is given by:

$$C_t = \left[ \omega \frac{1}{\gamma} C_{h,t}^{\frac{\gamma-1}{\gamma}} + (1 - \omega) \frac{1}{\gamma} C_{f,t}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \quad (2.3)$$

\(^6\)We can make the model more symmetric by assuming half of the traders lives in the home country and half of them lives in the foreign country. But this will only complicate the model without significantly changing the results.
where \( C_{t} = \left( \int_{0}^{1} C_{t}(i)^{\theta_{-1}} di \right)^{\frac{\theta}{\theta_{-1}}} \), \( C_{f,t} = \left( \int_{0}^{1} C_{f,t}(j)^{\theta_{-1}} dj \right)^{\frac{\theta}{\theta_{-1}}} \), and the weight \( \omega \in (0, 1) \) determines the home representative agent’s bias for the domestic composite good. Note that \( \theta \) is the elasticity of substitution between individual home (or foreign) goods and \( \gamma \) is the elasticity of substitution across home and foreign composite goods.

\( P_{t} \) is a consumption based price index for period \( t \), which is defined by:

\[
P_{t} = \left[ \omega P_{t}^{1-\gamma} + (1 - \omega)P_{f,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}
\]

(2.4)

where \( P_{h,t} = \left( \int_{0}^{1} P_{h,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \) and \( P_{f,t} = \left( \int_{0}^{1} P_{f,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \).

In each period every household is endowed with one unit of time, which is divided between leisure and work. His income is derived from the labor income \( W_{t}L_{t} \), profits from domestic goods producers (which is assumed to be owned by domestic households) \( \Pi_{t} \), interest received on domestic bonds \( B_{t}(1 + r_{t}) \) and lump-sum government transfer \( T_{t} \). Solving the household’s problem, the optimality conditions can be written as:

\[
\left( \frac{M_{t}}{P_{t}} \right)^{\epsilon} = \frac{(C_{t}^{H})^{\rho}}{1 - \frac{1}{1 + r_{t+1}}}
\]

(2.5)

\[
\eta L_{t}^{\psi} = \frac{W_{t}}{P_{t}(C_{t}^{H})^{\rho}}
\]

(2.6)

\[
\beta E_{t}\left( \frac{(C_{t}^{H})^{\rho}}{(C_{t+1}^{H})^{\rho}} \right) P_{t} = \frac{1}{1 + r_{t+1}}
\]

(2.7)

The first order conditions of the foreign households are entirely analogous, except that foreign household’s consumption is denoted by \( C_{t}^{*} \), as there is only one type of representative agent in the foreign country.

We assume firms have linear technologies, for each home good \( i \), \( y_{t}(i) = L_{t}(i) \). It is also assumed that, due to high costs of arbitrage for consumers, each individual monopolist can price discriminate across countries. Furthermore, as in Betts and Devereux (1996) and Chari, Kehoe and McGrattan (2002), we assume local currency pricing: firms set prices (separately) in the currencies of buyers. Finally, prices are assumed to be set one period in advance and cannot be revised until the following period. That is, the home monopolist sets \( P_{h,t}(i) \) optimally at the end of period \( t - 1 \), and these prices cannot be changed during time \( t \).

The technical appendix gives the derivation of the optimal pricing schedule of firms. The firms will just set the price so that it equals to a mark-up over the expected marginal cost and a risk premium term arising from the covariance of the firm’s profits with the marginal utility of consumption:

\[
P_{h,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1}[D_{t}W_{t}C_{t}]}{E_{t-1}[D_{t}C_{t}]} \quad P_{t}^{*} = \frac{\theta}{\theta - 1} \frac{E_{t-1}[D_{t}W_{t}C_{t}^{*}]}{E_{t-1}[D_{t}C_{t}^{*}]}
\]

(2.8)
\[ P_{f,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D^*_t W^*_t C_t]}{E_{t-1} [D^*_t S^*_t C_t]} \]  
\[ P^*_{f,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D^*_t W^*_t C^*_t]}{E_{t-1} [D^*_t C^*_t]} \]  
(2.9)

where \( D_t \) and \( D^*_t \) denote the pricing kernels households used to value date \( t \) profits. Because all home firms are assumed to be owned by the domestic households, it follows that in equilibrium \( D_t \) is the intertemporal marginal rate of substitution in consumption between time \( t - 1 \) and \( t \):

\[ D_t = \beta \frac{(C^H_t)^{\rho} P_{t-1}}{(C^H_{t-1})^{\rho} P_t} \]  
(2.10)

\( D^*_t \) is defined analogously. \( S_t \) is the nominal exchange rate at time \( t \).

The home government issues the local currency, has no expenditures, and runs a balanced budget every period. The nominal transfer is then given by:

\[ T_t = M_t - M_{t-1} \]  
(2.11)

The stochastic process that describes the evolution of the domestic money supply is:

\[ M^*_t = \mu_t M^*_{t-1} \]  
(2.12)

\[ \log(\mu_t) = \epsilon_{\mu,t} \]  
(2.13)

where \( \epsilon_{\mu,t} \sim N(0, \sigma_{\epsilon_{\mu}}^2) \) is a normally distributed random variable. The stochastic process of money supply in the foreign country is entirely analogous. Also, the home monetary shock and the foreign monetary shock are assumed to be independently distributed, i.e., \( \text{Cov}(\epsilon_{\mu}, \epsilon^*_{\mu}) = 0 \).

### 2.2 Foreign Exchange Market

#### 2.2.1 Foreign Exchange Traders

Following closely the work of De Long et al. (1990) and Jeanne and Rose (2002), the foreign exchange traders are modelled as overlapping generations of investors who decide how many one-period foreign nominal bonds to buy in the first period of their lives. Traders have the same taste, but differ in their abilities to trade in the foreign bond market. Some of them are able to form accurate expectations on risk and returns, while others have noisy expectation about future returns. The former are referred as the “rational trader” and the latter as the “noise traders”. Hereafter, the informed trader is denoted by a superscript \( I \) and the noise trader is denoted by a superscript \( N \).

Two specifications of the model are developed. In the first specification, the number of incumbent noise traders is exogenously determined. In the second one, the traders have to pay a fixed entry cost to trade on the foreign exchange market. The introduction of an entry cost...
helps to endogenize the noise component of the market. This makes the policy analysis possible as policy makers can affect the composition of traders through the entry cost.

In the foreign exchange market, at each period, a generation of foreign exchange traders is born. The continuum of the traders is indexed by \( i \in [0, 1] \). Assuming that in each generation of traders, \( N_I \) of them are rational traders, and \( 1 - N_I \) are noise traders. The timing of the model is illustrated in Figure 1.

![Figure 1: Timing of Model](image)

<table>
<thead>
<tr>
<th>t</th>
<th>t+1</th>
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<tbody>
<tr>
<td><strong>Action 1</strong></td>
<td><strong>Action 2</strong></td>
<td><strong>Action 3</strong></td>
</tr>
<tr>
<td>t foreign exchange trader ( i ) is born; Time ( t ) shocks and nominal interest rates are revealed; The time ( t ) born trader ( i ) decides if he should enter the foreign bond market.</td>
<td>He decides the number of foreign currency bonds ( B^<em><em>{h,t+1}(i) ) to purchase based on his expectation about future exchange rate ( S</em>{t+1} ). To finance his purchase, he borrows ( B^</em>_{h,t+1}(i)S_t ) from the home bond market.</td>
<td>Time ( t+1 ) exchange rate ( S_{t+1} ) is revealed, so the return of his investment in terms of home currency is realized, which is equal to ( S_{t+1}B^<em>_{h,t+1}(i)(1 + r^</em><em>{t+1}) ). He pays back the principle and interest of his borrowing ( (B^*</em>{h,t+1}(i)S_t(1 + r_{t+1})) ), gets the excess return, consumes, and dies.</td>
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Action 1: Time \( t \) foreign exchange trader \( i \) is born; Time \( t \) shocks and nominal interest rates are revealed; The time \( t \) born trader \( i \) decides if he should enter the foreign bond market.

Action 2: He decides the number of foreign currency bonds \( B^*_{h,t+1}(i) \) to purchase based on his expectation about future exchange rate \( S_{t+1} \). To finance his purchase, he borrows \( B^*_{h,t+1}(i)S_t \) from the home bond market.

Action 3: Time \( t+1 \) exchange rate \( S_{t+1} \) is revealed, so the return of his investment in terms of home currency is realized, which is equal to \( S_{t+1}B^*_{h,t+1}(i)(1 + r^*_{t+1}) \). He pays back the principle and interest of his borrowing \( (B^*_{h,t+1}(i)S_t(1 + r_{t+1})) \), gets the excess return, consumes, and dies.

Let \( \varphi^i_t \) denote the dummy variable characterizing the market-entry condition of period \( t \) born foreign exchange trader \( i \). If \( \varphi^i_t = 0 \), trader \( i \) will not enter the foreign bond market and if \( \varphi^i_t = 1 \), he will enter. At the beginning of period \( t \), trader \( i \) will enter the market as long as the expected utility of entering the market is higher than that of not entering:

\[
E^i_t(U^j_t \mid \varphi^i_t = 1) \geq E^i_t(U^j_t \mid \varphi^i_t = 0)
\]

(2.14)

A foreign exchange trader who has entered the foreign bond market maximizes a mean-
variance utility function:

\[
\max_{B^*_h,t+1(i)} E^i_t(C^T_{t+1}(i)) - \frac{a}{2} \text{Var}^i_t(C^T_{t+1}(i))
\] (2.15)

Subject to

\[
P_{t+1}C^T_{t+1}(i) = [B^*_h,t+1(i)(1 + r^*_t+1)S_{t+1} - B^*_h,t+1(i)S_t(1 + r_t+1)] - P_{t+1}c_i - TT(i)
\] (2.16)

where \(B^*_h,t+1(i)\) denotes the amount of one-period foreign currency bonds held by trader \(i\) from time \(t\) to time \(t + 1\), \(a\) is the absolute risk aversion coefficient; \(c_i\) reflects the costs associated with entering the foreign bond market for trader \(i\). \(TT(i)\) is the trader \(i\)'s transaction cost of trading foreign-currency bond. It is assumed that

\[
TT(i) = \tau \frac{B^*_h,t+1(i)^2}{2} S_t
\] (2.17)

where \(\tau > 0\) is the rate of the transaction tax on foreign bond trading, or, in other words, the ‘Tobin tax’ rate. Here, the transaction cost is modelled as a convex transaction cost because a linear cost would imply that trader \(i\) will gain when selling foreign bonds.\(^8\) Note that the transaction cost is in foreign currency as trader \(i\) is trading foreign bonds. Therefore, in terms of domestic currency, it is \(\tau \frac{B^*_h,t+1(i)^2}{2} S_t\).

The entry costs may include information costs for investment in the foreign bond market and other costs when investing abroad. To formalize this heterogeneity, here we follow the specification used by Jeanne and Rose (2002). Rational traders are assumed to have a larger stock of knowledge about the economy and thus, do not need to invest in the acquisition of information. Their entry costs are therefore zero. For noise traders, they do not have a natural ability to acquire and process the information about the economy and therefore have to pay a positive entry cost.

Although the preferences of all noise traders are the same, the noise traders are assumed to be distinguished from each other by their entry costs. Without loss of generality, the noise traders are indexed by increasing entry costs:

\[
c_i = \bar{c} \left\{ \frac{i}{1 - N_I} \right\}^\alpha \text{ for } i \in [0, 1 - N_I]
\] (2.18)

\(^7\)Ex post, the net return on the foreign exchange traders’ position could be negative. That is, the traders can have negative consumption. One way to fix this problem is to assume that each period traders enter the market with an endowment which is large enough to ensure their consumption can never be negative. So the foreign exchange traders will never default. Nevertheless, the existence of this endowment will not affect the optimal bond holding problem of the traders. Thus, we do not model it explicitly.

\(^8\)Also, in reality the transaction cost in foreign exchange market is usually convex in the amount of foreign currency traded.
where $\alpha > 0$ is the curvature parameter and $\bar{c}$ is the parameter characterizing the scale or level of the entry cost of the noise traders. Thus, the noise trader at the left end of the continuum ($i$ near 0) tends to have a lower entry cost and the noise trader towards the right end of the continuum ($i$ near $1 - N_t$) has a higher entry cost.

### 2.2.2 Optimal demand for foreign bonds

Once traders have decided to enter the market, the optimal demand for foreign bonds of each type of traders can be derived. Substituting Equations 2.16, 2.17 into Equation 2.15, gives:

$$
\max_{B^*_h,t+1(i)} \left\{ E_t \left[ B^*_h,t+1(i) S_t (1 + r_{t+1}) \over P_{t+1} \right] - c_i - \tau - \frac{B^*_h,t+1(i)^2}{2P_{t+1}} S_t \right\} - \frac{a}{2} Var_t \left[ B^*_h,t+1(i) S_t (1 + r_{t+1}) \over P_{t+1} \right] \rho_{t+1} \right] 
$$

where

$$
\rho_{t+1} = \left[ \frac{S_t (1 + r_{t+1})}{S_{t+1}(1 + r_{t+1})} - 1 \right] \tag{2.20}
$$

is the excess return.

We now discuss the information structure of traders. Specifically, we make the following assumptions about the subjective distribution over $\rho_{t+1}$. The rational traders can predict $\rho_{t+1}$ correctly; while the noise traders cannot predict the future excess return correctly. That is, for informed traders:

$$
E_t^I[\rho_{t+1}] = E_t[\rho_{t+1}] \tag{2.21}
$$

$$
Var_t^I[\rho_{t+1}] = Var_t[\rho_{t+1}] \tag{2.22}
$$

For noise traders, following the work of De Long et al. (1990), we assume:

$$
E_t^N[\rho_{t+1}] = E_t[\rho_{t+1}] + v_t \tag{2.23}
$$

$$
Var_t^N[\rho_{t+1}] = Var_t[\rho_{t+1}] \tag{2.24}
$$

$$
Var(v_t) = \lambda Var(s_t) \quad \text{where } \lambda \in (0, +\infty) \tag{2.25}
$$

where $v_t$ is assumed to be i.i.d and normally distributed with zero mean. $\lambda$ can be considered as a parameter characterizing the relative magnitude of noise traders’ erroneous beliefs to exchange rate volatility.

From Equations 2.23 and 2.21, it can be seen that, compared to the rational trader’s expectation, the noise traders’ expectation of $\rho_{t+1}$ based on time $t$ information is biased from

---

Note that $P_{t+1}$ is known at $t$ because we assume that price are set one period ahead. Meanwhile, $B^*_h,t+1$, $S_t$, and $r_{t+1}$ are known at $t$ by assumptions.
the true conditional expectation by a random error. Nevertheless, noise traders can correctly forecast the conditional variance of the exchange rate. From Equation 2.25, another assumption is made that the unconditional variance of $v_t$ is proportional to the unconditional variance of the exchange rate itself. This assumption helps to tie down the scale of the volatility of noise traders’ erroneous beliefs.\footnote{The logic behind this assumption is that the bias in noise traders' expectation must be related to the volatility of the exchange rate itself, otherwise noise traders might expect the future exchange rate to be volatile even under a fixed exchange rate regime.} Solving Equation 2.19, the optimal bond holding of trader $i$ is given by:

$$B_{h,t+1}^{i,*} = \frac{E_t[\rho_{t+1}]}{(1+r_{t+1})} + a \frac{S_t}{r_{t+1}} (1 + r_{t+1}) Var_t[\rho_{t+1}]$$

(2.26)

Therefore, informed traders and noise traders differ in their optimal bond holding. Also, from Equation 2.26, the lower the expected excess return, the higher the risk (excess return volatility) and the risk coefficient, the less bond traders (both rational traders and noise traders) will hold. Thus, the traders account for risk when taking positions on assets. It can also be seen that the Tobin tax reduces the bond trading of both types of traders. This is quite intuitive, as foreign exchange traders will tend to trade less foreign currency bonds when there is a tax on transactions.

At the margin, the return from enlarging one’s position in an asset that is mispriced (the expected excess return) is offset by the additional price risk (the volatility of the excess return) and transaction cost (the Tobin tax) that must be borne.

### 2.2.3 Equilibrium condition of the foreign exchange market

**Analysis with no entry costs** We first analyze a simple case where $\bar{c} = 0$. As shown in Appendix A, if there is no entry cost, traders will always choose to enter the market. This is because the transaction cost is convex in the bonds traded, traders can always choose to hold a small amount of foreign bonds and get a positive expected utility, regardless of how large $\tau$ is. Thus, in this case all noise traders will enter the market and the noise component of the market is exogenously determined by the number of existing noise traders $(1 - N_I)$ on the market. So the aggregate demand for foreign bonds by foreign exchange traders of the home country can be denoted as:

$$B_{h,t+1}^* = N_I B_{h,t+1}^{I,*} + (1 - N_I) B_{h,t+1}^{N,*}$$

$$= \frac{E_t[\rho_{t+1}]}{(1+r_{t+1})} + a \frac{S_t}{r_{t+1}} (1 + r_{t+1}) Var_t[\rho_{t+1}]$$

(2.27)
\[ E_t [\rho_{t+1}] + (1 - N_I)v_t - a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(\rho_{t+1})B_{h,t+1}^* - \frac{\tau}{(1 + r_{t+1})} B_{h,t+1}^* = 0 \quad (2.28) \]

**Endogenous entry of noise traders** We now endogenize the composition of traders who enter the market in each period by introducing positive entry costs for noise traders. The entry decision for informed traders is trivial. They bear no entry cost and always enter the foreign bonds market in equilibrium. A noise trader, however, enters if and only if Equation 2.14 is satisfied. As shown in Appendix A, for trader \( i \), this condition takes the form:

\[ c_i \leq \left[ \frac{E_t^N(\rho_{t+1})^2}{2a \text{Var}_t(\rho_{t+1}) + 2\tau \frac{P_{t+1}}{S_t(1+r_{t+1})^2}} \right]^\frac{1}{\alpha} \equiv GB_t^N \quad (2.29) \]

where \( GB_t^N \) is the gross benefit of entry for noise traders. It increases with the expected excess return and decreases with conditional time \( t + 1 \) exchange rate volatility and the Tobin tax.

Let \( c_i^* = GB_t^N \) be the cut-off value of entry cost. From Equation 2.29, for noise trader \( i \), if \( c_i \leq c_i^* \), \( \varphi_i^t = 1 \); if \( c_i > c_i^* \), \( \varphi_i^t = 0 \). The number of incumbent noise traders \( n_t \) is then given by:

\[ n_t = \min \left[ \left( \frac{c_i^*}{c} \right)^\frac{1}{\alpha}, 1 \right] (1 - N_I) = \min \left\{ \left[ \frac{E_t^N(\rho_{t+1})^2}{2a \text{Var}_t(\rho_{t+1}) + 2\tau \frac{P_{t+1}}{S_t(1+r_{t+1})^2}} \right]^\frac{1}{\alpha}, 1 \right\} (1 - N_I) \quad (2.30) \]

Apparently, the number of active noise traders on the market increases with the square of the expected excess return and decreases with conditional time \( t + 1 \) exchange rate volatility and the number of existing noise traders, and decreases with the entry cost, the risk aversion coefficient \( a \), the Tobin tax \( \tau \), and the excess return volatility. The economic intuition behind Equation 2.30 is as follows. The presence of more active noise traders creates higher expected excess return and incentives for other noise traders to enter the market, however, the extra volatility brought about by their entry will reduce the gross benefit of entry for noise traders. In equilibrium, the two effects balance and no more noise traders will enter.

Substituting Equation 2.30 into \( B_{h,t+1}^* = N_I B_{t+1}^I + n_t B_{h,t+1}^N \), we can derive the equilibrium condition of the foreign bond market when the entry decision of traders is endogenized:

\[ E_t(\rho_{t+1}) + \frac{n_t v_t}{N_I + n_t} - a \frac{S_t(1 + r_{t+1})}{P_{t+1}(N_I + n_t)} \text{Var}_t(\rho_{t+1})B_{h,t+1}^* - \frac{\tau}{(1 + r_{t+1})(N_I + n_t)} B_{h,t+1}^* = 0 \quad (2.31) \]

where \( n_t \) is given by Equation 2.30. Equations 2.28 and 2.31 represent the interest parity conditions in this economy. Note that the uncovered interest parity does not hold in this model. The last three terms in Equations 2.28 and Equation 2.31 show the deviation from the uncovered interest parity when noise traders are present in the market. This deviation consists of three
parts. The first part is the expectation error of the noise traders, which increase when there are more incumbent noise traders. The second part is the risk premium term, since the foreign exchange traders are risk averse. Besides the expectation error term and the risk premium term, there is an extra term that comes from the transaction tax. Even in the absence of noise traders, the second and third term still exists.\footnote{But as shown in the literature, these terms are not volatile enough to explain the exchange rate volatility.}

\section{Equilibrium Condition}

Equilibrium for this economy is a collection of 28 sequences \((P_t, P^*_t, P^*_h,t, P^*_f,t, P^*_h,t, C_t, C_t^T, C_t^H, C_t^r, C_{h,t}, C_{h,t}^*, C_{f,t}, C_{f,t}^*, S_t, r_t, r^*_t, D_t, D_t^*, W_t, W_t^*, B_t, B_t^*, B^*_h,t, L_t, L_t^*, n_t, \rho_{t+1})\) satisfying 28 equilibrium conditions. They include the six household optimality conditions (Equations 2.5, 2.6, 2.7 and their foreign counterparts), the definition of the price indexes (Equation 2.4 and its foreign analogy), the definition of the pricing kernel (Equation 2.10 and its foreign analogy), the interest parity conditions (Equation 2.28 or Equation 2.31), the definition of \(n_t\) (Equation 2.30) for the endogenous entry case, the definition of excess return \(\rho_{t+1}\) (Equation 2.20), the four individual demand equations, the four pricing conditions, and the four market clearing conditions for the bonds and goods markets:

\begin{align*}
B_{t+1} &= S_t B^*_{h,t+1} \forall t \quad (2.32) \\
B^*_t + B^*_h,t &= 0 \quad (2.33) \\
L_t &= C_{h,t} + C^*_{h,t} \quad (2.34) \\
L^*_t &= C_{f,t} + C^*_{f,t} \quad (2.35)
\end{align*}

Finally, the budget constraint of the foreign exchange traders.

\begin{align*}
P_tC_t^T &= B^*_h,t (1 + r_t^*) S_t - B^*_h,t S_{t-1} (1 + r_{t-1}) - \tau \frac{B^*_h,t^2}{2} S_{t-1} \quad \text{Exogenous Entry} \quad (2.36) \\
P_tC_t^T &= \left[ B^*_h,t (1 + r_t^*) S_t - B^*_h,t S_{t-1} (1 + r_{t-1}) \right] - P_t \sum_{i=0}^{n_t} c_i - \tau \frac{B^*_h,t^2}{2} S_{t-1} \quad \text{Endogenous Entry} \quad (2.37)
\end{align*}

And the home country aggregate consumption equation:

\begin{align*}
C_t &= C_t^H + C_t^T + \tau \frac{B^*_h,t^2}{2} S_{t-1} \quad \text{Exogenous Entry} \quad (2.38) \\
C_t &= C_t^H + C_t^T + \sum_{i=0}^{n_t} c_i + \tau \frac{B^*_h,t^2}{2} S_{t-1} \quad \text{Endogenous Entry} \quad (2.39)
\end{align*}
Then the above two equations, the budget constraints of the home households 2.2, Equation 2.32 and its one-period lag can be combined to get the national budget constraint of the home country:

$$P_t C_t = W_t L_t + \Pi_t + S_t B^*_h, t(1 + r_t^*) - S_t B^*_{h, t+1}$$  \hspace{1cm} (2.40)

where $\Pi_t = \omega \left[ (P_{h, t} - W_t) \left( \frac{P_{h, t}}{P_t} \right)^{-\gamma} C_t + (P^*_h S_t - W_t) \left( \frac{P^*_h}{P_t} \right)^{-\gamma} C^*_t \right]$.

3 Model Solution

The detailed model solution, including the approximation of the system of equation and the derivation of solution, is given in the technical appendix. Here we will just outline the solution method.

The equilibrium conditions of this economy can be divided into conditions with variance term $Var_t(\rho_{t+1})$ (the interest parity conditions) and conditions without the variance term. Since the second-order terms are important for understanding dynamics of the economy, especially for the exchange rate, we will solve the model by the following steps.

First, we will take a second-order approximation of the equilibrium conditions with variance terms: the interest parity conditions (Equation 2.28 and Equation 2.31). By second-order approximation, we can keep the variance term and expectation error terms. Given the second-order approximation of these equations, it can be shown that it is only necessary to solve a first-order approximation of the other equilibrium conditions of the model. This is because the second-order approximations of Equations 2.28 and 2.31 only contain terms in the square and cross products of the variables of the model. Thus second order accurate solutions for these terms can be obtained from a first-order solution to the equilibrium conditions without variance terms.

Therefore, the second step is to log-linearize other equilibrium conditions. The point of approximation is the non-stochastic, symmetric steady state (as described in Appendix B), where net foreign assets are zero, all prices are equal, and the exchange rate is unity. Given

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12If we only use first-order approximation to approximate the interest parity conditions, then the expectation error terms and the variance terms will disappear. We could not analyze the endogenous entry case and the impact of Tobin tax. In other words, policy analysis is not possible.

13This method is firstly used by Devereux and Sutherland (2006) to solve a model where there is portfolio holding. In our paper, the interest parity condition is analogous to their portfolio conditions.

14The financial market are incomplete in our model since the home and foreign household only have access to non-state contingent domestic currency nominal bonds. If there are no foreign exchange traders, then there is a unit root in the net foreign assets in this kind of model. However, when the foreign exchange traders are present, the net foreign assets are zero at the steady state. Please see Appendix B for detail.
the approximated system of equilibrium conditions, the final step is to solve deviations of the exchange rate and other macroeconomic variables from their \( t - 1 \) expectations in terms of exogenous money supply shocks and the expectation error shocks. And then we can see if exchange rate disconnect can be achieved in this model.

Hereafter, \( \hat{x}_t = \log(X_t) - \log(\bar{X}) \), \( dX_t = X_t - \bar{X} \), where \( \bar{X} \) is the non-stochastic steady state value of variable \( X_t \). And let \( x_{t+j} = x_{t+j}^* - E_{t-1}(x_{t+j}^*) \), \( j \geq 0 \) denote the deviation of a variable from its date \( t - 1 \) expectation.\(^{15}\)

### 3.1 Model 1: Exogenous Entry

Following the first two steps outlined above, we can get the approximated system of equations. Then, the log-linearized home household’s budget constraint minus its \( t - 1 \) expectation gives:\(^ {16}\)

\[
c_t^H - c_t^* + \frac{2}{PC}dB_{t+1} = \bar{s}_t
\]

(3.1)

The right-hand side of Equation 3.1 represents the relative wealth effect of an unanticipated shock to the exchange rate through firms’ profits. This relative wealth increase will be spread between an increase in relative home consumption and net foreign assets accumulation.

Using the log-linearized goods market clearing condition and Equation 3.1, we get:

\[
(c_t^H - c_t^*) + \frac{\sigma}{\bar{r}}E_t(c_{t+1}^H - c_{t+1}^*) = \bar{s}_t
\]

(3.2)

where \( \sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma} \). This equation implies that the wealth effect of an increase in exchange rate will be spread between increase in current relative consumption and expected period \( t + 1 \) relative consumption.

Then, using the log-linearized intertemporal optimality equations and the interest parity condition, we may obtain the consumption-based interest parity condition:

\[
E_t(c_{t+1}^H - c_{t+1}^*) = (c_t^H - c_t^*) - \frac{1}{\rho} \left\{ \bar{s}_t - (1 - N_l)v_t + \left[ a \frac{(1 + \bar{r})S}{P} \text{Var}_t(s_{t+1}) + \frac{\tau}{1 + \bar{r}} \right] dB_{n,t+1}^* \right\}
\]

(3.3)

This equation implies expected consumption growth in the home country decreases in response to an unanticipated exchange rate depreciation, since it generates an unanticipated real depreciation, and therefore reduces the home country’s real interest rate.

\(^{15}\)Hereafter, the curvature parameter of entry costs \( \alpha \) is set to be equal to 1. The model can be easily extended to the case where \( \alpha > 1 \) or \( 0 < \alpha < 1 \), and the main results will not change. Also, it is assumed that the elasticity of the money demand \( \epsilon = 1 \), which is close to the estimate reported in Mankiw and Summers (1986).

\(^{16}\)See the technical appendix for details of derivations of equations.
Finally, the relation between relative money supply and relative consumption can be derived from the money demand equations:\(^\text{17}\)

\[ \tilde{m}_t - \tilde{m}_t^* = \rho (\tilde{c}_t^H - \tilde{c}_t^* ) \]  

(3.4)

Putting Equations 3.1, 3.2, 3.3 and 3.4 together, we can get a system of equilibrium conditions that characterizes \( \{ \tilde{s}_t, \tilde{c}_t - \tilde{c}_t^*, dB_{h,t+1}^* \} \). This gives us the solution of \( \tilde{s}_t \) in terms of \( v_t \) and monetary shocks.

\[ \tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) \frac{1 + \frac{\sigma}{\rho} + \frac{\phi \text{Var}(s_{t+1}^\tilde{\tau}) + \frac{\xi}{\rho} \tau}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau}}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau} (1 - N_I) v_t \]  

(3.5)

where

\[ \phi = \frac{a (1 + \bar{r}) \ddot{S} \ddot{C} \sigma}{2 \bar{r}} \quad \xi = \frac{\ddot{P} \ddot{C} \sigma}{2 (1 + \bar{r}) \bar{r}} \]  

(3.6)

From Equation 3.5, the variance of the future exchange rate deviation, \( \text{Var}_t(s_{t+1}^\tilde{\tau}) \) can be solved.\(^\text{18}\) Let \( \text{Var}_t(s_{t+1}^\tilde{\tau}) \equiv V_s \), \( V_s \) is given by the following implicit function:

\[ V_s = \frac{\left(1 + \frac{\sigma}{\rho} + \frac{\phi \text{Var}(s_{t+1}^\tilde{\tau}) + \frac{\xi}{\rho} \tau}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau}\right)^2}{1 - \left(\frac{\sigma}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau}\right)^2 (1 - N_I)^2 \lambda} \left[ \text{Var}(\tilde{m}_t) + \text{Var}(\tilde{m}_t^*) \right] \]  

(3.7)

Note that the coefficient \( \phi \) is associated with the risk-aversion of traders. The higher the risk aversion coefficient, the lower will be the exchange rate volatility. For both types of traders, their aversion to risk prevents exchange rate volatility from increasing too much. To see this, it can be shown that as long as \( \rho > 1 \), holding other parameters constant, the numerator \( \frac{1 + \frac{\sigma}{\rho} + \frac{\phi \text{Var}(s_{t+1}^\tilde{\tau}) + \frac{\xi}{\rho} \tau}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau}}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau} \) is decreasing in \( V_s \) and the denominator \( 1 - \left(\frac{\sigma}{\rho + \frac{\sigma}{\rho} + \phi \text{Var}(s_{t+1}^\tilde{\tau}) + \xi \tau}\right)^2 (1 - N_I)^2 \lambda \) is increasing in \( V_s \).

Similarly, it can easily be shown that given other parameters, the numerator decreases in \( \tau \), the rate of transaction tax, while the denominator increases in \( \tau \). So this implies that the Tobin tax will reduce exchange rate volatility. The exactly relationship between \( \tau \) ahd \( V_s \) will be discussed in Section 4.

Can the exchange rate display ‘excess volatility’ in this model? When \( \rho = 1 \), the coefficient in front of \( (\tilde{m}_t - \tilde{m}_t^*) \) is exactly 1 in Equation 3.5. Therefore, with no noise traders, the exchange rate volatility will be equal to that of the fundamentals. If noise traders are present on the

\(^{17}\)Notice that \( \tilde{m}_t = \epsilon_{\rho, r} \), and \( \tilde{m}_t^* = \epsilon_{\rho, r}^* \). We use \( \tilde{m}_t \) and \( \tilde{m}_t^* \) for notational convenience.

\(^{18}\)Since \( \tilde{s}_t \) is linear in \( \tilde{m}_t \), \( \tilde{m}_t^* \) and \( v_t \) and the monetary shocks and expectation error shocks are normally distributed with zero mean and constant variance, \( \text{Var}_t(s_{t+1}^\tilde{\tau}) = \text{Var}(s_{t+1}^\tilde{\tau}) = \text{constant} \equiv V_s \).
market, the exchange rate volatility may be much higher than the fundamental volatility, even when $\rho = 1$.

What are the responses of macroeconomic fundamentals such as consumption, labor and wage to the exogenous monetary shocks and expectation error shocks? From the log-linearized goods market clearing condition, labor supply condition and the money demand condition,

$$
\tilde{w}_t = \frac{\psi}{2\rho}(\tilde{m}_t + \tilde{m}_t^*) + \tilde{m}_t \\
\tilde{l}_t = \frac{1}{2\rho}(\tilde{m}_t + \tilde{m}_t^*)
$$

(3.8)

$$
\tilde{c}_t = \frac{1}{\rho} \tilde{m}_t \\
\tilde{c}_t^* = \frac{1}{\rho} \tilde{m}_t^*
$$

(3.9)

Therefore, the volatilities of the macroeconomic fundamentals are only decided by the volatility of the relative monetary shock and consumer preferences, but not by the volatility of the expectation error and the number of incumbent noise traders in the market.

Note that from Equations 3.1 and 3.4, the net foreign assets are given by:

$$
dB_{t+1} = \frac{\tilde{P}C}{2} [(\tilde{s}_t - \frac{1}{\rho}(\tilde{m}_t - \tilde{m}_t^*))]
$$

(3.10)

Thus, the volatility of the net foreign assets will be affected by the number of incumbent noise traders. But as shown in Section 4, their volatility is much smaller when compared to exchange rate volatilities.

### 3.2 Model 2: Endogenous Entry

The endogenous entry case is similar to the exogenous entry case except for the interest parity equation. So following the same method, we can get the solution to the endogenous entry model, and the derivation is entirely analogous to Equation 3.5. The technical appendix gives details:

$$
\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) \left\{ \frac{1 + \frac{\sigma}{\bar{r}} + \phi' Var_t(s_{t+1}) + \xi' \tau}{\rho + \frac{\sigma}{\bar{r}} + \phi' Var_t(s_{t+1}) + \xi' \tau} + \frac{\sigma}{\bar{r} + \phi' Var_t(s_{t+1}) + \xi' \tau} \right\} \frac{1}{\bar{P}C} \tilde{P} \tilde{C} \tilde{n}_t \tilde{v}_t
$$

(3.11)

where

$$
\tilde{n}_t \approx \min \left\{ \frac{[E_t(s_{t+1}) - \tilde{s}_t + \tilde{v}_t]^2}{2aVar_t(s_{t+1}) + 2\tau \frac{\rho}{S(1+\bar{r})\bar{r}}} \frac{1 - N_I}{\tilde{c}}, 1 - N_I \right\}
$$

(3.12)

$$
\phi' = \frac{a(1 + \bar{v})S \bar{C} \sigma}{2N_I} \frac{\sigma}{\bar{r}} \\
\xi' = \frac{\bar{P}C}{2(1 + \bar{r})N_I} \frac{\sigma}{\bar{r}}
$$

(3.13)

Analogous to the exogenous entry case, when $\rho = 1$ and no noise traders are present ($n_t = 0$), $\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*)$, exchange rate volatility will be identical to that of the fundamental. When there are noise traders in the foreign exchange market, the exchange rate may diverge significantly
from the fundamental values. Note that now the Tobin tax not only affects the exchange rate volatility through the $\xi'\tau$ term as in the exogenous entry case, but also affects the exchange rate volatility through $n_t$, the number of incumbent noise traders. Therefore, the impact of the Tobin tax on exchange volatility depends on the market structure and the interaction of the Tobin tax with other trading cost on foreign exchange market. Section 4 will discuss the implication of the Tobin tax further. Finally, the expression for net foreign assets, consumption, labor, and wage are exactly the same as in the exogenous case.

4 Results

Equations 3.5 and 3.11 are too complicated to be solved analytically, so the numerical undetermined coefficient method described in the technical appendix is used to solve for $\tilde{s}_t$, $V_s$ and $E_s$.

Table 1 gives the parameter values that are used in the numerical simulation. We choose $\beta = 0.94$, which produces a steady state real interest rate of six percent, about the average long-run real return on stocks. The parameters $\eta$ and $\psi$ are set so that the elasticity of labor supply is 1 and the time devoted to work is one quarter of the total time in the steady state. The business cycle literature has a wide range of estimates for the curvature parameter $\rho$. Chari, Kehoe, and McGrattan (2002) set $\rho = 5$ to generate a high volatility of the real exchange rate. In our model, a high exchange rate volatility can be obtained without high risk aversion of households, so it is set to equal 2. For the final goods technology parameters, the elasticity of substitution between domestic produced goods $\theta$ is set to 11 following Betts and Devereux (2000). This gives a wage-price mark-up of about 1.1, which is consistent with the finding of Basu and Fernald (1994). The elasticity of substitution between home goods and foreign goods $\gamma$ is set to be 1.5, following Chari, Kehoe, and McGrattan (2002) and Backus, Kehoe, and Kydland (1994). Note that, other parameters, such as the money supply process, number of informed traders on the market, and entry costs, are not fully calibrated as the purpose of this paper is just qualitative analysis.

In the first subsection, we will analyze the implication of noise traders in explaining exchange rate disconnect. Then we will consider the endogenous entry of noise traders in the second subsection. We then explore the effect of the Tobin tax on exchange rate volatility in the last subsection.
TABLE 1
Parameter Values

<table>
<thead>
<tr>
<th>Exogenous Case</th>
<th>Endogenous Case*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td>Foreign exchange traders</td>
</tr>
<tr>
<td>$\beta = 0.94$, $\rho = 2$, $\epsilon = 1$, $\eta = 58.2$, $\psi = 1$</td>
<td>Other parameters in the endogenous case are the same as in the exogenous case.</td>
</tr>
<tr>
<td>Final goods technology</td>
<td>$\theta = 11$, $\gamma = 1.5$, $\omega = 0.5$</td>
</tr>
<tr>
<td>Money Growth Process</td>
<td>$\text{corr}(\varepsilon_{\mu}, \varepsilon_{\mu}^<em>) = 0$, $\sigma_{\varepsilon_{\mu}}^2 = \sigma_{\varepsilon_{\mu}^</em>}^2 = 0.01$</td>
</tr>
<tr>
<td>Foreign exchange traders</td>
<td>$\bar{c} = 0$, $N_t \in [0, 1]$, $\lambda = 1.5$</td>
</tr>
<tr>
<td>Steady State Values</td>
<td>$\mu_{ss} = \mu_{ss}^* = 1$, $M_{ss} = M_{ss}^* = 2$</td>
</tr>
</tbody>
</table>

4.1 Exogenous Case

We first solve for the exogenous entry case. Table 2 illustrates the results of simulations. The first ten rows show changes in volatilities of the exchange rate and net foreign assets when the number of noise traders increases from 0 to 1. The last three rows report volatilities of the macroeconomic fundamental variables, given the calibrated parameter values.

From Table 2, three important findings are: First, the exchange rate volatility increases when the number of noise traders increases, while volatilities of macroeconomic fundamentals remain constant. Moreover, the exchange rate volatility is much higher than that of the macroeconomic fundamentals. Second, from the functional form of $\tilde{s}_t$ listed in Table 2, the impact of fundamental monetary shocks on the exchange rate (coefficient of $\tilde{m}_t$ or $\tilde{m}_t^*$) decreases in the number of noise traders. Meanwhile, the effect of expectation error on exchange rate (coefficient of $v_t$) increases when more noise traders are present on the market. Third, the exchange rate volatility is higher when the magnification coefficient $\lambda$ increases.

Therefore, the critical implication is that “disconnection” does exist between the exchange rate and the macroeconomic fundamentals in this model. The presence of noise traders in the foreign exchange market, combined with local currency pricing, generates a degree of exchange rate volatility that may be much higher than that of the underlying fundamental shocks. In other words, the “exchange rate disconnect puzzle” may be explained by the approach suggested in this paper.

Why the disconnect puzzle can be explained in such a model? This is mainly because the presence of expectation errors and the risk premium term of noise traders drives wedges between
home and foreign real interest rates, which breaks the link between an exchange rate shock and the home relative real interest rates (home relative expected future consumption) implied by the interest rate parity condition.

To see this, we can compare the consumption-based interest rate parity conditions with and without noise traders:

\[ E_t(c^H_{t+1} - c^*_t) = (c^H_t - c^*_t) - \frac{1}{\rho} s_t, \text{ w/o noise traders} \]  
\[ E_t(c^H_{t+1} - c^*_t) = (c^H_t - c^*_t) - \frac{1}{\rho} \left( s_t - (1 - N_I) v_t + \left[ a \frac{(1 + \bar{r}) S}{P} Var_t(s^*_t) + \frac{\tau}{1 + \bar{r}} \right] dB^*_h,t+1 \right), \text{ w/ noise traders} \]  

The interest parity conditions imply that a depreciation of home currency today will reduce the relative real interest rate in the home country and change the path of consumption, so that the current home consumption will increase, relative to the expected future consumption. But to get a disconnection between consumption and exchange rate, we need to have a small change in current consumption and a big change in exchange rate. From Equation 4.1, this implies that the expected future consumption has to drop a lot. This is inconsistent with the wealth effect of a depreciation (as shown by Equation 3.2, when the home currency depreciates, the domestic currency value of foreign sales will increase, which increases both current and future home relative consumption). Therefore, with no noise traders, the only possible way to explain the difference between the exchange rate volatility and the fundamental volatility is by introducing a high value of \( \rho \), which is exactly the mechanism emphasized in Chari, Kehoe, and McGrattan (2002).

With noise traders, we can see from Equation 3.3 that now a large increase of exchange rate and a small change in current consumption do not necessarily imply a large drop of expected future consumption, because the presence of the expectation errors and the risk premium term of noise traders also drive wedges between the home and foreign real interest rates. This could be called the “level effects” created by the noise traders.\(^{19}\)

The presence of noise traders also creates a “volatility effect”, since the volatility of \( v_t \) itself is proportional to the exchange rate volatility. This property “magnifies” the response of the exchange rate to the expectation error of noise traders. When the nominal exchange rate volatility increases, so does the expectation error volatility, which further increases the exchange rate volatility until the system reaches an equilibrium where \( Var(v_t) = \lambda Var(s_t) \). Thus, the larger \( \lambda \), the higher is the exchange rate volatility.\(^{19}\)

\(^{19}\)This is first pointed out by Devereux and Engel (2002).
Therefore, volatile exchange rates can be obtained in this model. Still, why is the high volatility not transferred to other macroeconomic variables (except for the net foreign assets)? Normally, there are two channels through which the exchange rate affects other macroeconomic variables: expenditure-switching effects and wealth effects. Since the prices of the import goods are assumed to be sticky in terms of the local currency, the relative price of home-produced goods to foreign-produced goods will remain unchanged when the exchange rate changes. Therefore, the expenditure-switching channel is completely shut down in this model.

How about the wealth effect? From Equation 3.1, the increase in wealth that comes from an unexpected depreciation will be spread between an increase in relative home consumption and the net foreign assets accumulation. From Equation 3.4, however, the increase in relative home consumption is limited by the relative money shocks due to the real balance effect. Therefore, the net foreign assets will absorb most of the wealth increase. This is actually shown in Table 2, when the volatility of the nominal exchange rate increases, so does the volatility of net foreign assets. However, the magnitude of the volatility of the net foreign asset and the expected future relative consumption are small quantitatively, especially when compared to that of the exchange rate. That implies the wealth effect are also quite small quantitatively. From Equation 3.2,

$$E_t(c^H_{t+1} - c^\ast_{t+1}) = \bar{r} \sigma [\tilde{s}_t - (c^H_t - c^\ast_t)]$$

(4.3)

It can be seen that the volatility of the change in expected future consumption is quantitatively small because $\frac{\bar{r}}{\sigma}$ is small given reasonable parameter values.\(^{20}\) The economic intuition is that the consumption-smoothing behavior of infinitely lived households limits the wealth effect in this model. When a shock leading to an exchange rate depreciation occurs, the households increase their holdings of net foreign assets. This increase will be spread over many future periods because households want to smooth their future consumption. The increase in the expected consumption of next period is then quite small. Therefore, the more risk averse are the households (the higher $\rho$), the bigger will be $\sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma}$ (suppose that $\gamma > 1$), and the smaller will be the wealth effect.\(^{21}\)

\(^{20}\)For current calibration, $\frac{\bar{r}}{\sigma} = 0.06/1.4 = 0.0429$. Recall that $\sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma}$, so as long as the elasticity of substitution between home and foreign goods $\gamma$ is greater then 1, $\sigma$ is greater than 1. Thus, $\frac{\bar{r}}{\sigma} < 0.06$.

\(^{21}\)Devereux and Engel (2002) emphasize what are essential for a general equilibrium model to explain the exchange rate disconnect puzzle theoretically. So they assume a specific production and distribution structure to remove wealth effects completely. Nevertheless, as illustrated above, wealth effects are quantitatively small in standard infinitely horizon DSGE models, given reasonable parameter values. And this result applies to the Devereux and Engel (2002)'s model as well. In this paper, the model is solved numerically and we focus on quantitative results. Therefore, we did not assume any specific currency-trading mechanism to remove wealth
Moreover, in the monetary model of exchange rate without noise traders, the monetary shocks lead to movements in both macroeconomic fundamentals and exchange rates, as shown by the following equations:\textsuperscript{22}

\[
\tilde{s}_t = \frac{1 + \frac{\sigma}{\rho}}{\rho} (\tilde{m}_t - \tilde{m}_t^*) \quad (4.4)
\]

\[
\tilde{c}_t^{H} - \tilde{c}_t^{*} = \frac{1}{\rho} (\tilde{m}_t - \tilde{m}_t^*) \quad (4.5)
\]

Therefore, it generically predicts a strong comovement and a high and positive correlation between the exchange rate and relative consumption.\textsuperscript{23} From empirical evidence, however, there is no clear path in the observed cross-correlation. Chari, Kehoe and McGrattan (2002) find that this correlation is negative for U.S. and Europe while it ranges between small and positive to somewhat negative for other country pairs.

Our model, however, does not predict a strong comovement of the exchange rate and relative consumption. The functional form of \(\tilde{s}_t\) listed in Table 2 shows that the exchange rate can move even when the realizations of the fundamentals shocks are equal to zero. Further more, as shown by the sixth column of Table 2, the cross-correlation between the exchange rate and relative consumption decreases when more noise traders are present on the foreign exchange market. Intuitively, this is because our model setting separates the foreign exchange market from the rest of the model, insuring that the expectation error creates a source of fluctuation which only affects exchange rate, but not macroeconomic fundamentals. Therefore, both expectation error shocks and monetary shocks will cause exchange rate fluctuations, but macroeconomic fundamentals can only be affected by monetary shocks. So our model can predict a small and positive correlation between the exchange rate and relative consumption.

\textbf{4.2 Endogenous Entry}

The exogenous entry specification gives important implications of the model, however, a natural question is what can monetary authorities do to get rid of the excess volatility in the nominal exchange rate? So in this subsection, we consider introducing entry cost to endogenize the entry of noise traders, which will help to evaluate the implications of policies that target the non-fundamental risk.

Table 3 illustrates simulation results of the endogenous entry case: First, the exchange rate disconnection still holds in this specification. Second, increasing the entry cost \(\bar{c}\) (within a rea-effects in the model setting.

\textsuperscript{22}With no traders on the foreign exchange market, Equation 3.5 could be rewritten as Equation 4.4.

\textsuperscript{23}For example, in Chari, Kehoe and McGrattan (2002) the correlation is equal to 1.
sonable domain of $\bar{c}$ will reduce the exchange rate volatility. The first finding is not surprising. As in the exogenous entry case, the presence of noise traders generates a wedge between home and foreign real interest rates. The only difference is that now the number of incumbent noise traders is endogenously decided, and as is the expectation error part. Nevertheless, this does not alter any of the theoretical analysis in Section 4.1. This wedge creates the “level effects” and the “volatility effects”, which in turn imply a degree of exchange rate volatility that is much higher than the fundamental volatility. Meanwhile, the expenditure-switching effect is eliminated because of the LCP pricing behavior. The wealth effect is quantitatively small because of households’ consumption smoothing behavior in an infinite horizon model. Therefore, the exchange rate volatility will not be transferred to the macroeconomic fundamentals except for net foreign assets.

The second finding is quite interesting and has important policy implications. Although the model is complicated and can only be solved numerically, this result is quite intuitive. The higher the entry cost, the fewer noise traders will enter the market and therefore fewer noise components will be present. Thus, it shows that exchange rate policies that aim at eliminating the non-fundamental risk can be justified theoretically. It also suggests possible approaches monetary authorities may apply to reduce the excess exchange rate volatility. They can discourage the entrance of noise traders by increasing the entry cost, or ‘educate’ the market to reduce the number of noise traders on the foreign exchange market.

4.3 The effect of the Tobin Tax

Tobin (1978) and Eichengreen, Tobin and Wyplosz (1995) suggest that an international transaction tax on purchases and sales of foreign exchange would be one way to “throw sand in the wheels of super-efficient financial vehicles”. They argue that a transaction tax might diminish excess volatility. Even a small transaction tax would deter the fast round trip into a foreign money market.

A Tobin tax is different from the entry cost we analyzed above. First, it is a common cost for both rational and noise traders. Second, it is not a fixed cost, but increases with the amount of foreign currency bond traded. In this subsection, we will analyze the implication of the Tobin tax on exchange rate volatility in our model.

As shown in Appendix A, without entry cost, all traders will always choose to enter the foreign exchange market no matter how large the Tobin tax is. Therefore, we will explore the impact of the Tobin tax on exchange rate volatility in two cases. In the exogenous entry case,
we focus on the Tobin tax only. In the endogenous entry case, we assume that noise traders have to pay two costs to trade in the foreign exchange market: the transaction cost (a Tobin tax) and a fixed information cost as in the previous subsection. However, the informed traders only need to pay a Tobin tax. The analysis of the second case will help to understand if the interaction of two costs will affect the effect of the Tobin tax.

**Exogenous entry case** Solving Equation 3.7 numerically can give us the relationship between the exchange rate volatility $V_s$ and the Tobin tax $\tau$, which is given in Table 4. It can be seen that the higher the rate of the transaction tax, the lower is the exchange rate volatility.

Intuitively, this is because the introduction of the transaction tax reduces the bond trading. In our model, when an exchange rate change occurs, the real balance effect prevents the current consumption from increasing/decreasing more than the changes in the relative real money supply, so the bond holding of households will absorb most of the wealth effect caused by the exchange rate change. If the bond trading is deterred by the transaction tax, in equilibrium, exchange rate movements will also be constrained, which implies a smaller exchange rate volatility.

As shown by Equation 2.17, all bond trading are subject to the Tobin tax. Therefore, the increase of the Tobin tax will discourage both the destabilizing and stabilizing transaction on the foreign exchange market. Thus, in a partial equilibrium model, sometimes it is hard to identify the impact of Tobin tax on exchange rate volatility as it will affect both type of traders.24 But as shown above, in a full-fledged macroeconomics general equilibrium model, the decrease of bond trading caused by increases of the Tobin tax will limit the exchange rate changes and thus lead to a decrease of exchange rate volatility. This feature is unique in a open economy macroeconomics GE model.

**Endogenous entry case** For the endogenous entry model, solving Equation 3.11 numerically, we find that exchange rate volatility also decreases in the transaction tax, as in the exogenous case. The results are given in Table 4.

In the endogenous entry case, the transaction cost reduces the exchange rate volatility through two channels. First, as in the exogenous case, it reduces the bond trading of both types of traders, which in turn decreases the exchange rate volatility. Second, as shown by

---

24For example, in Jeanne and Rose (2002)'s model, if a Tobin tax is considered as in Equation 2.17, it will lead to decreases of bond trading, but will have no impact on exchange rate volatility in the no entry cost case. This is because without entry cost, Tobin tax itself will not affect the noise component as all traders will always choose to enter the foreign exchange market no matter how large the Tobin tax is.
Equation 2.29, the Tobin tax reduces the gross benefit of entry for noise traders, which consequently reduces the noise component of the foreign exchange market (see Equation 3.12). Therefore, the mechanism through which the transaction cost affects the exchange rate volatility is different when the noise component on the market is endogenously determined. The effect of the Tobin tax will also be different. This can be seen from Table 4, for the same level of increase in $\tau$, the decrease in the exchange rate volatility in the endogenous entry case is larger than that in the exogenous entry case. This finding has important policy implications. It shows that the impact of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

Since James Tobin proposed the Tobin tax in 1974, the debate about the Tobin tax in foreign exchange market concentrates on its feasibility and the “distorting” effects it might have as a tax. Few papers, however, investigate whether or not a Tobin tax is effective in reducing financial market volatility. In this paper we show that a Tobin tax can reduce exchange rate volatility without affecting macroeconomic fundamentals in a DSGE model. We also show that the impact of a Tobin tax depends on the presence of other fixed entry costs.

The impact of Tobin tax on exchange rate volatility naturally leads to a question about optimal tax rates, especially in a general equilibrium model like ours. Nevertheless, we focus on the exchange rate disconnect puzzle in this paper. Thus, as shown above, in our current framework the exchange rate volatility has little effect on macroeconomic variables such as consumption and labor supply, which implies that the current specification is not an appropriate framework for discussion of the optimal Tobin tax rate. However, if the model specification is changed to include the mechanism through which exchange rate volatility may have substantive impacts on firms’ and households’ behavior, the approach proposed in our paper can be applied to study the optimal Tobin tax rate.\footnote{For example, if there exists balance sheet effect for firms, decreases of exchange rate volatility may be welfare improving. Meanwhile, international bond trading is helpful for the risk sharing across countries. A Tobin tax, which reduces the bond trading, will deter the risk sharing as well. Therefore, a general equilibrium model incorporating noise traders and these features can be used to evaluate the household’s welfare under different tax rates. For reasons of scope, we feel that an analysis of this question could not be pursued in the current paper. In our view, this issue is important enough to deserve investigation in a separate paper.}
5 Conclusions and Subsequent Research

In this paper we present a model of exchange rate determination which combines the new open economy macroeconomics approach and the noise trader approach for exchange rate behavior. Therefore, this paper emphasizes the interaction of the macroeconomic fundamentals of exchange rate and the microstructure channel through which exchange rates are determined.

The major findings of this paper are: 1. Models that take both the macroeconomic and microeconomic factors of exchange rate determination into consideration can explain the “exchange rate disconnect puzzle”. 2. The exchange rate volatility caused by irrational market behavior or non-fundamental shocks can be reduced by policies. We analyze two kinds of policies. One focuses on the entry cost of noise traders, while the other is a ‘Tobin tax’ type of policy. We find that both policies can reduce the exchange rate volatility. However, the effect of the Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

Subsequent research should focus on the policy implication of this model. If the real exchange rate volatility is primarily affected by non-fundamental factors, then would the fixed exchange rate regime or a single currency area welfare-dominate the flexible exchange regime? This model could also be used to evaluate the welfare implications of policies such as the Tobin tax or other policies that discourage the entry of noise traders. These policies are discussed widely, but due to the lack of a welfare-based model which can explain exchange rate volatility and its relationship with macroeconomic fundamentals, they have not been evaluated on a welfare basis.

Although this model can help to explain the exchange rate disconnect puzzle, it could not explain other exchange rate dynamics, such as the persistence of real exchange rates. To explain this, more persistent price setting or ‘sticky’ information of traders may be needed.
\[ s_t = 0.9562 \tilde{m}_t - 0.9562 \tilde{m}_t^* + 0.0912 v_t \]

<table>
<thead>
<tr>
<th>No. of Noise Traders</th>
<th>( \tilde{s}_t = 0.9561 \tilde{m}_t - 0.9561 \tilde{m}_t^* + 0.1824 v_t )</th>
<th>( \tilde{s}_t = 0.9559 \tilde{m}_t - 0.9559 \tilde{m}_t^* + 0.2736 v_t )</th>
<th>( \tilde{s}_t = 0.9557 \tilde{m}_t - 0.9557 \tilde{m}_t^* + 0.3645 v_t )</th>
<th>( \tilde{s}_t = 0.9553 \tilde{m}_t - 0.9553 \tilde{m}_t^* + 0.4553 v_t )</th>
<th>( \tilde{s}_t = 0.9546 \tilde{m}_t - 0.9546 \tilde{m}_t^* + 0.5455 v_t )</th>
<th>( \tilde{s}_t = 0.9532 \tilde{m}_t - 0.9532 \tilde{m}_t^* + 0.6344 v_t )</th>
<th>( \tilde{s}_t = 0.9494 \tilde{m}_t - 0.9494 \tilde{m}_t^* + 0.7191 v_t )</th>
<th>( \tilde{s}_t = 0.9351 \tilde{m}_t - 0.9351 \tilde{m}_t^* + 0.7831 v_t )</th>
<th>( \tilde{s}_t = 0.9024 \tilde{m}_t - 0.9024 \tilde{m}_t^* + 0.8047 v_t )</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>0.0183</td>
<td>0.00%</td>
<td>2.3978E-04</td>
<td>1.0000</td>
<td>0.0183</td>
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<td>0.1</td>
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<td>12.58%</td>
<td>3.7264E-04</td>
<td>0.9422</td>
<td>0.0192</td>
<td>5.23%</td>
<td>2.9500E-04</td>
<td>0.9747</td>
<td>0.0189</td>
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<tr>
<td>0.2</td>
<td>0.0228</td>
<td>24.77%</td>
<td>5.0124E-04</td>
<td>0.8948</td>
<td>0.0265</td>
<td>44.85%</td>
<td>7.1322E-04</td>
<td>0.8301</td>
<td>0.0231</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0329</td>
<td>80.00%</td>
<td>1.0844E-03</td>
<td>0.7441</td>
<td>0.0206</td>
<td>12.58%</td>
<td>3.7264E-04</td>
<td>0.9422</td>
<td>0.0198</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0459</td>
<td>150.75%</td>
<td>3.2055E-02</td>
<td>0.6295</td>
<td>0.0228</td>
<td>24.77%</td>
<td>5.0124E-04</td>
<td>0.8948</td>
<td>0.0211</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0803</td>
<td>339.33%</td>
<td>3.8217E-03</td>
<td>0.4737</td>
<td>0.0265</td>
<td>44.85%</td>
<td>7.1322E-04</td>
<td>0.8301</td>
<td>0.0231</td>
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<tr>
<td>0.6</td>
<td>0.0732</td>
<td>109.25%</td>
<td>3.2055E-02</td>
<td>0.6295</td>
<td>0.0459</td>
<td>150.75%</td>
<td>3.2055E-02</td>
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<td>0.0307</td>
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<tr>
<td>0.7</td>
<td>0.2183</td>
<td>1094.09%</td>
<td>1.1787E-02</td>
<td>0.2830</td>
<td>0.0803</td>
<td>339.33%</td>
<td>3.8217E-03</td>
<td>0.4737</td>
<td>0.0385</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5695</td>
<td>3014.60%</td>
<td>3.2055E-02</td>
<td>0.6295</td>
<td>0.2183</td>
<td>1094.09%</td>
<td>1.1787E-02</td>
<td>0.2830</td>
<td>0.0538</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9024 \tilde{m}_t - 0.9024 \tilde{m}_t^* + 0.8047 v_t</td>
<td>0.0915</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.0025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

\[ V \text{ar}(\tilde{c}_t) = 0.0025 \]

\[ V \text{ar}(\tilde{w}_t) = 0.0163 \]

\[ V \text{ar}(\tilde{l}_t) = 0.0013 \]

---

\( \lambda = 1/1.5, a = 2, \tau = 0.0001 \)
### TABLE 3

<table>
<thead>
<tr>
<th>Endogenous Case ($\lambda = 1.5$, $\alpha = 2$, $\tau = 0.0001$)</th>
<th>$N_I = 0.1$</th>
<th>$\bar{c} = 0.15$</th>
<th>$\bar{c} = 0.25$</th>
<th>$\bar{c} = 0.15$</th>
<th>$\bar{c} = 0.25$</th>
<th>$\bar{c} = \infty$</th>
<th>$\bar{c} = \infty$</th>
<th>$\bar{c} = \infty$</th>
<th>$\bar{c} = \infty$</th>
<th>$\bar{c} = \infty$</th>
<th>$\bar{c} = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{c} = 0.1$</td>
<td>0.7303</td>
<td>0.3688</td>
<td>0.1180</td>
<td>0.0167</td>
<td>0.2482</td>
<td>0.0722</td>
<td>0.0297</td>
<td>0.0171</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{c} = 0.2$</td>
<td>0.1215</td>
<td>0.0981</td>
<td>0.0031</td>
<td>0</td>
<td>0.1742</td>
<td>0.1665</td>
<td>0.1750</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>$\bar{c} = 0.4$</td>
<td>0.0431</td>
<td>0.0238</td>
<td>0.0121</td>
<td>0.0173</td>
<td>0.0212</td>
<td>0.0201</td>
<td>0.0193</td>
<td>0.0173</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{c} = 0.8$</td>
<td>0.2938</td>
<td>0.2834</td>
<td>0.2627</td>
<td>0</td>
<td>0.3065</td>
<td>0.2886</td>
<td>0.2600</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{c} = 0.99$</td>
<td>0.1493</td>
<td>0.1400</td>
<td>0.1257</td>
<td>0</td>
<td>0.0076</td>
<td>0.0071</td>
<td>0.0065</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$V_{\text{ar}}(\hat{s}t) = 0.7363$ 0.3688 0.1189 0.0167 $V_{\text{ar}}(\hat{s}t) = 0.2482$ 0.0722 0.0297 0.0171

$V_{\text{ar}}(\hat{d}B_{t+1}) = 4.22E-02$ 2.11E-02 6.20E-03 2.11E-04 $V_{\text{ar}}(\hat{d}B_{t+1}) = 1.41E-02$ 4.02E-03 1.56E-03 2.20E-04

$Mean(\hat{n}) = 0.1739$ 0.1215 0.0981 0.0031 $Mean(\hat{n}) = 0.1742$ 0.1665 0.1750 0

Here we only consider a reasonable range of $\bar{c}$: $\bar{c} \in (0, 0.25)$, as the steady state consumption in this model is 0.25.
<table>
<thead>
<tr>
<th>Tobin Tax</th>
<th>(\Var(\tilde{s}_t)) (Exogenous Case, (\lambda = 1.5, a = 2))</th>
<th>(\Var(\tilde{s}_t)) (Endogenous Case, (\lambda = 1.5, a = 2, \bar{c} = 0.1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau = 0.0001)</td>
<td>(N_I = 0) = 0.5695 &lt;br&gt; (N_I = 0.1) = 0.2183 &lt;br&gt; (N_I = 0.2) = 0.0803 &lt;br&gt; (N_I = 0.3) = 0.0458 &lt;br&gt; (N_I = 0.4) = 0.0329 &lt;br&gt; (N_I = 0.5) = 0.0265</td>
<td>(N_I = 0.001) = 1.2710 &lt;br&gt; (N_I = 0.1) = 0.7363 &lt;br&gt; (N_I = 0.2) = 0.2482 &lt;br&gt; (N_I = 0.3) = 0.0571 &lt;br&gt; (N_I = 0.4) = 0.0311</td>
</tr>
<tr>
<td>(\tau = 0.5%)</td>
<td>(N_I = 0) = 0.5627 &lt;br&gt; (N_I = 0.1) = 0.2150 &lt;br&gt; (N_I = 0.2) = 0.0798 &lt;br&gt; (N_I = 0.3) = 0.0457 &lt;br&gt; (N_I = 0.4) = 0.0328 &lt;br&gt; (N_I = 0.5) = 0.0264</td>
<td>(N_I = 0.001) = 1.2625 &lt;br&gt; (N_I = 0.1) = 0.7254 &lt;br&gt; (N_I = 0.2) = 0.2307 &lt;br&gt; (N_I = 0.3) = 0.0480 &lt;br&gt; (N_I = 0.4) = 0.0280 &lt;br&gt; (N_I = 0.5) = 0.0204</td>
</tr>
<tr>
<td>(\tau = 1%)</td>
<td>(N_I = 0) = 0.5620 &lt;br&gt; (N_I = 0.1) = 0.2138 &lt;br&gt; (N_I = 0.2) = 0.0792 &lt;br&gt; (N_I = 0.3) = 0.0455 &lt;br&gt; (N_I = 0.4) = 0.0327 &lt;br&gt; (N_I = 0.5) = 0.0264</td>
<td>(N_I = 0.001) = 1.2536 &lt;br&gt; (N_I = 0.1) = 0.7143 &lt;br&gt; (N_I = 0.2) = 0.2115 &lt;br&gt; (N_I = 0.3) = 0.0397 &lt;br&gt; (N_I = 0.4) = 0.0249 &lt;br&gt; (N_I = 0.5) = 0.02</td>
</tr>
<tr>
<td>(\tau = 1.5%)</td>
<td>(N_I = 0) = 0.5582 &lt;br&gt; (N_I = 0.1) = 0.2115 &lt;br&gt; (N_I = 0.2) = 0.0787 &lt;br&gt; (N_I = 0.3) = 0.0453 &lt;br&gt; (N_I = 0.4) = 0.0327 &lt;br&gt; (N_I = 0.5) = 0.0263</td>
<td>(N_I = 0.001) = 1.2448 &lt;br&gt; (N_I = 0.1) = 0.7030 &lt;br&gt; (N_I = 0.2) = 0.1917 &lt;br&gt; (N_I = 0.3) = 0.0324 &lt;br&gt; (N_I = 0.4) = 0.0226 &lt;br&gt; (N_I = 0.5) = 0.0197</td>
</tr>
<tr>
<td>(\tau = 2%)</td>
<td>(N_I = 0) = 0.5544 &lt;br&gt; (N_I = 0.1) = 0.2092 &lt;br&gt; (N_I = 0.2) = 0.0782 &lt;br&gt; (N_I = 0.3) = 0.0451 &lt;br&gt; (N_I = 0.4) = 0.0326 &lt;br&gt; (N_I = 0.5) = 0.0263</td>
<td>(N_I = 0.001) = 1.2360 &lt;br&gt; (N_I = 0.1) = 0.6915 &lt;br&gt; (N_I = 0.2) = 0.1709 &lt;br&gt; (N_I = 0.3) = 0.0269 &lt;br&gt; (N_I = 0.4) = 0.0210 &lt;br&gt; (N_I = 0.5) = 0.0193</td>
</tr>
<tr>
<td>(\tau = 2.5%)</td>
<td>(N_I = 0) = 0.5506 &lt;br&gt; (N_I = 0.1) = 0.2070 &lt;br&gt; (N_I = 0.2) = 0.0776 &lt;br&gt; (N_I = 0.3) = 0.0450 &lt;br&gt; (N_I = 0.4) = 0.0325 &lt;br&gt; (N_I = 0.5) = 0.0262</td>
<td>(N_I = 0.001) = 1.2270 &lt;br&gt; (N_I = 0.1) = 0.6798 &lt;br&gt; (N_I = 0.2) = 0.1486 &lt;br&gt; (N_I = 0.3) = 0.0240 &lt;br&gt; (N_I = 0.4) = 0.0205 &lt;br&gt; (N_I = 0.5) = 0.0190</td>
</tr>
</tbody>
</table>
A Entry Condition of Traders with Tobin Tax

This appendix derives the entry condition (Equation 2.29) for noise traders to enter the foreign bond market. The noise trader \( i \) will enter the foreign bond market if and only if Equation 2.14 holds.

If trader \( i \) does not enter the market, his expected utility is given by:

\[
E_i^t(U_i^t \mid \varphi_i^t = 0) = 0 \tag{A.1}
\]

Therefore, with only transaction cost, trader \( i \) will enter the market if and only if:

\[
E_i^t \left\{ \max_{B_{h,t+1}(i)} \left\{ E_i^t \left[ B_{h,t+1}(i) S_t(1 + r_{t+1}) \rho_{t+1} - \tau \frac{B_{h,t+1}(i)^2}{2P_{t+1}} S_t \right] \right\} \right\} \geq 0 \tag{A.2}
\]

Substituting \( B_{h,t+1} = E_i^t[\rho_{t+1}] \) into the above equation gives:

\[
E_i^t \left\{ \frac{E_i^t[\rho_{t+1}]}{(1 + r_{t+1}) + a S_t(1 + r_{t+1}) Var_t(\rho_{t+1})} \frac{S_t(1 + r_{t+1})}{P_{t+1}} \rho_{t+1} - \frac{\tau}{2} \left[ \frac{E_i^t[\rho_{t+1}]}{(1 + r_{t+1}) + a S_t(1 + r_{t+1}) Var_t(\rho_{t+1})} \right]^2 S_t \right\} \geq 0 \tag{A.3}
\]

Or

\[
\zeta \left\{ \frac{S_t(1 + r_{t+1})}{P_{t+1}} \left[ \frac{\tau}{(1 + r_{t+1}) + a S_t(1 + r_{t+1}) Var_t(\rho_{t+1})} \right] + a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) Var_t(\rho_{t+1}) \right\} - \frac{\tau}{2} \frac{S_t}{P_{t+1}} - a Var_t(\rho_{t+1}) \frac{S_t(1 + r_{t+1})}{P_{t+1}} \geq 0 \tag{A.4}
\]

where

\[
\zeta = \frac{[E_i^t(\rho_{t+1})]^2}{2 \left[ \frac{\tau}{(1 + r_{t+1}) + a S_t(1 + r_{t+1}) Var_t(\rho_{t+1})} \right]^2} \geq 0 \tag{A.5}
\]

It can be shown that the terms in the big bracket of Equation A.4 are equal to:

\[
\tau \frac{S_t}{P_{t+1}} + a \left( \frac{S_t(1 + r_{t+1})}{P_{t+1}} \right)^2 Var_t(\rho_{t+1}) \geq 0 \tag{A.6}
\]

Therefore, regardless of how large the Tobin tax rate (\( \tau \)) is, the traders will always enter the foreign bond market.
When the noise traders has to pay two costs to trade on the foreign exchange market, for noise trader $i$, he will enter the market if and only if:

$$E_i \left\{ \max_{B_{h,t+1}^i(i)} \left\{ E_t^i \left[ \frac{B_{h,t+1}^i(i)S_t(1+r_{t+1})}{P_{t+1}} - \frac{B_{h,t+1}^i(i)^2}{2} - c_i \right] - \frac{a}{2} Var_t^i \left[ \frac{B_{h,t+1}^i(i)S_t(1+r_{t+1})}{P_{t+1}} \right] \right\} \right\} \geq 0$$  \hspace{1cm} (A.7)

Substituting $B_{h,t+1}^i(i) = E_N^\tau \left[ \frac{\tau}{(1+r_{t+1})} + a \frac{S_t}{P_{t+1}} (1+r_{t+1}) Var_t(\rho_{t+1}) \right]$ into the above equation, it can be shown that Equation A.7 is equivalent to:

$$\zeta_N \left\{ 2 \frac{S_t(1+r_{t+1})}{P_{t+1}} \left[ \frac{\tau}{(1+r_{t+1})} + a \frac{S_t}{P_{t+1}} (1+r_{t+1}) Var_t(\rho_{t+1}) \right] - \frac{c_i}{\zeta_N} - \tau \frac{S_t}{P_{t+1}} - a Var_t(\rho_{t+1}) \left[ \frac{S_t(1+r_{t+1})}{P_{t+1}} \right]^2 \right\} \geq 0$$  \hspace{1cm} (A.8)

where

$$\zeta_N = \frac{\left[ E_t^N(\rho_{t+1}) \right]^2}{2 \left[ \frac{\tau}{(1+r_{t+1})} + a \frac{S_t}{P_{t+1}} (1+r_{t+1}) Var_t(\rho_{t+1}) \right]^2} \hspace{1cm} (A.9)$$

It is easy to show that terms in the big bracket of Equation A.8 are equal to

$$\tau \frac{S_t}{P_{t+1}} + a \left[ \frac{S_t(1+r_{t+1})}{P_{t+1}} \right]^2 Var_t(\rho_{t+1}) - \frac{c_i}{\zeta_N} \hspace{1cm} (A.10)$$

Therefore, we may establish that Equation A.8 is equivalent to:

$$\varphi_i^t = 1 \iff \tau \frac{S_t}{P_{t+1}} + a \left[ \frac{S_t(1+r_{t+1})}{P_{t+1}} \right]^2 Var_t(\rho_{t+1}) - \frac{c_i}{\zeta_N} \geq 0 \hspace{1cm} (A.11)$$

Therefore, we could get the following entry condition for noise trader $i$:

$$\varphi_i^t = 1 \iff c_i \leq \frac{\left[ E_t^N(\rho_{t+1}) \right]^2}{2a Var_t(\rho_{t+1}) + 2\tau \frac{P_{t+1}}{S_t(1+r_{t+1})^2}} \equiv GB \hspace{1cm} (A.12)$$

### B A Symmetric Steady State

In a non-stochastic steady state, all shocks are equal to zero. Hereafter, steady state values are marked by overbars.

As the consumption is constant at the steady state, the steady state world interest rate $\bar{r}$ is tied down by the intertemporal optimality equation (Equation 2.7):

$$\bar{r} = \bar{r}^* = \frac{1-\beta}{\beta} \hspace{1cm} (B.1)$$

From the pricing equation, at the steady state, all the prices are equal and steady state exchange rate $\bar{S} = 1$. Then the steady state excess return $\bar{\rho} = \frac{\bar{S}(1+r^*)}{\bar{S}(1+r)} - 1 = 0$. From Equation 2.26, we will have \(^{26}\)

$$B_{h}^\tau(i) = 0 \hspace{0.5cm} \forall i \in [0,1] \hspace{1cm} (B.2)$$

\(^{26}\)Note that since $v_t = 0$, only rational traders are present on the market.
The economic intuition behind B.2 is that traders are not going to hold foreign bonds because they know that the excess return will be zero, and thus no trade takes place. The only way that no trade will occur in equilibrium is for the uncovered interest parity to hold.

Therefore, from the bond market clearing condition, net foreign assets are zero.

\[
\bar{B} = \bar{B}^* = 0
\]  

(B.3)

The steady state values of other variables are straight forward. Since \(\bar{B} = 0\), a closed-form solution exists for the steady state, in which the countries have identical outputs, consumption and real money holdings:

\[
\bar{L} = \bar{L}^* = \bar{C} = \bar{C}^* = \left( \frac{\theta \eta}{\theta - 1} \right)^{\frac{1}{\rho + \psi}}
\]  

(B.4)

\[
\frac{\bar{M}}{\bar{P}} = \frac{\bar{M}^*}{\bar{P}^*} = \left( \frac{\bar{C}^\rho}{1 - \beta} \right)^{\frac{1}{2}}
\]  

(B.5)
References


