Optimal Monetary Policy with Vertical Production and Trade*

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Abstract

This paper explores the optimal monetary policy response to domestic and foreign technology shocks in an open economy with vertical structure of production and trade. We find that any stage-specific productivity shock in one country may have a trans-border spillover effect on the other country via the vertical trade. So when choosing the optimal monetary rules, each monetary authority should respond to both home and foreign productivity shocks. Also, the flexible exchange rate can not replicate the flexible price equilibrium, even under producer currency pricing, due to price stickiness in multiple stages. Meanwhile, the exchange rate in such an environment will be more stable than that of an economy without vertical structure of production and trade. We also find that the existence of a trans-border spillover effect depends on the currencies of price setting. Finally, vertical trade may affect the value of exchange rate flexibility under PCP and LCP setting.

JEL classification: F3, F4

Keywords: optimal monetary policy, sticky price, vertical production and trade.

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1 Introduction

The debate on optimal monetary policy has been at the heart of international macroeconomics for many years. Friedman (1953) and later Mundell (1961) argue that the flexible exchange rate can act as an efficient mechanism for dealing with country-specific shocks when the adjustment of domestic price levels is sluggish. But recent studies of monetary policy in utility-based open economy sticky-price models have reached varying conclusions about the desirability of flexible exchange rates and the way monetary authorities respond to foreign shocks.

Obstfeld and Rogoff (2002) argue that a monetary policy in which the monetary authorities respond solely to their domestic shocks delivers the best possible outcome, and the flexible exchange rate can replicate the flexible price equilibrium. Devereux and Engel (2003), however, show that the optimal monetary policy and exchange rate regime choice critically depends on the currency of export pricing. If prices are set in the currency of producers (PCP), and the pass-through from exchange rate to consumer prices is complete, then the flexible exchange rate is a central part of the optimal monetary policy. But if the prices are set in the currency of buyers (LCP) and do not respond to movements in exchange rates, then the monetary authorities should keep the exchange rate fixed, so they will respond to both home and foreign shocks, and the optimal monetary policy cannot replicate the flexible price equilibrium[1]. Therefore, given the optimal monetary policy, the welfare under LCP is always lower than that of PCP structure.

Tille (2002) emphasizes the importance of the nature and sources of shocks on the optimal monetary policy design. He shows that the value of exchange rate flexibility is much smaller when shocks are sector-specific and argues that the sectoral structure of the economy and the source of shocks significantly affect the international monetary policy and its welfare implication.

Therefore, these literatures suggest that the presence of local currency pricing (or incomplete exchange rate pass-through) and sectoral shocks may significantly change the

[1]Corsetti and Pesenti (2001) have a similar conclusion. They analyze how the degree of exchange rate pass-through to prices affects the optimal monetary policy and show that the optimal monetary regime is pegged exchange rate in the extreme case where the exchange rate has no impact on consumer price.

existing wisdom on international monetary policy based on PCP pricing.

A notable feature of these literatures, however, is that they focus on an environment where all the trade in goods between countries occurs in one stage. In reality, countries can trade not only finished goods but also intermediate goods, even in more stages. We will follow Hummels et al. (1998) and use the term "vertical trade" to describe this vertical structure of production and trade. More specifically, vertical trade occurs when a country uses imported intermediate goods as an input to produce export goods. This definition captures the idea that countries are linked sequentially in the production of final goods.

Hummels et al. (1998) analyze data from 10 OECD countries and find a strong statistical correlation between the increase in the share of vertical trade in total trade and the rise of the share of trade in GDP. The increase in the vertical trade is found to account for more than 25 percent of the increase in the total trade in most OECD countries. In some smaller countries, such as Canada and Netherlands, the share of vertical trade in total trade approaches 50 percent. Feenstra(1998), Hummels et al. (2001), and Yi (2003) all argue that the vertical structure has been an increasingly important feature of today's global production and trade. Huang and Liu (2001) show that a vertical chain of production can generate a different monetary transmission mechanism in a closed economy.

Yet the vertical structure of production and trade has been remarkably overlooked in the new open economy macroeconomics literature. Obstfeld and Rogoff (2000) and Devereux and Engel (2005) introduce two stages of production, but the international trade in their models is limited to only intermediate goods. Huang and Liu (2004a) try to reconcile the controversy of the welfare consequence of *unilateral* monetary expansion under PCP and LCP pricing by modelling multiple stages of production and trade into a new open economy macroeconomic framework. Nevertheless, none of the above literatures analyzes the *optimal monetary policy* with vertical trade.

Intuitively, vertical structure of production and trade will influence the international transmission mechanism of productivity shocks and thus affect the way monetary authorities respond to country-specific and stage-specific productivity shocks. Also, vertical trade implies that terms of trade in multiple stages of production have to be adjusted in response to stage-specific shocks, so it may change the desirability of the flexible exchange rate in optimal monetary policy. Therefore, this paper tends to explore optimal monetary policy in an open economy with vertical production and trade.

To address this question, we introduce two stages of production and trade into a standard two-country general equilibrium model with sticky prices. There are two vertical stages of production in each country: one is the finished goods stage; the other one is the intermediate goods stage. Each stage in each country has a stage-specific productivity shock. There exists vertical trade in this model, as the production of finished goods requires both domestic and imported intermediate goods. To compare with the existing literatures on optimal monetary policy, we first maintain the assumption that firms set prices in PCP in both stages. Our model is simple enough to be solved analytically, so the policy evaluation will be based on rigorous welfare analysis. Nevertheless, it incorporates the main feature of the vertical production and trade we want to emphasize in this paper.

Our main findings from this benchmark model are as follows: First, with vertical trade, any stage-specific productivity shock in one country has a trans-border spillover effect on the other country via vertical production and trade. This effect changes the way monetary authority reacts to the other country's productivity shock. Each monetary authority should respond positively and partly to both home and foreign productivity shocks. This is quite different from Obstfeld and Rogoff (2002) and Devereux and Engel (2003), where the optimal monetary policy based on PCP requires monetary authorities to respond only to domestic shocks[2]. Second, a vertical production structure generates multiple price stickiness. Thus, the flexible exchange rate cannot adjust terms of trade in both stages to the efficient level simultaneously. So unlike the argument by Friedmand (1953) and later Obstfeld and Rogoff (2002), the flexible price equilibrium cannot be replicated by flexible exchange rate in our model, even in the situation with PCP pricing and complete exchange rate pass-through. Finally, we find that the exchange rate volatility in this economy is much lower than would be obtained in an economy without vertical

[2]Devereux and Engle (2003) show that optimal monetary policy requires a fixed exchange rate under LCP pricing; thus, home and foreign monetary authorities respond identically to country-specific productivity shocks. Intuitively, this is because the exchange rate could not adjust terms of trade under LCP pricing, but fixing the exchange rate can ensure international risk-sharing. Therefore, the mechanism leading to positive response to foreign shocks is completely different in our model.

structure of production and trade, as multiple stages of production and *trade* lead to a more integrated global economy.

From the above findings, several interesting questions can be raised. First, does the trans-border spillover effect depend on the pricing behavior of firms? Second, because the exchange rate could not adjust terms of trade in both stages to the efficient level simultaneously, will vertical trade structure change the value of exchange rate flexibility under PCP and LCP pricing? To answer these questions, we extend the model to allow for LCP pricing in the intermediate goods stage.

We find that the existence of the trans-border spillover effect depends not only on the vertical trade structure but also on the the currencies of pricing setting. This is because the spillover effect works though the relative price adjustment caused by exchange rate changes. Foreign productivity shock in other stages can affect the home relative demand in one stage only when the terms of trade can be adjusted in that stage. So the spillover effect exists only when prices are set under PCP pricing in both stages.

From the welfare analysis, we find that compared to the benchmark pure PCP model, the optimal monetary policy in the model with LCP in intermediate stages can deliver a higher welfare, when the productivity shocks in two stages are negatively correlated. This is different from the welfare implication of one-stage models like Devereux and Engel (2003) and Corsetti and Pesenti (2004), where the welfare given by optimal monetary policy under LCP pricing is always lower than that under PCP structure. Intuitively, this is because the value of exchange rate flexibility under PCP is lower when productivity shocks are negatively correlated. So when the terms of trade adjustment are eliminated in the intermediate goods stage, the exchange rate can fully adjust the terms of trade in the finished goods stage to its efficient level, which actually improves welfare for the whole economy. This result illustrates the impact of vertical trade on the value of exchange rate flexibility under different pricing structures.

This paper is related to Devereux and Engel (2003). We adopt their approach to derive optimal monetary rules. Devereux and Engel (2005) also build a new open economy macro-economic model with the intermediate goods trade. They develop a view of the exchange rate policy as a trade-off between the desire to ensure international risk-sharing and the need to facilitate relative price adjustment, and optimal nominal exchange rate volatility will reflect these competing objectives. In our paper, as emphasized, we introduce vertical structure of production and *trade* into the new open macroeconomic framework. Therefore, we focus on the difference in the international transmission mechanism of productivity shocks and the limitation of the exchange rate as an adjustment mechanism for nominal rigidity in such an environment.

As to the emphasis on multiple stages of production and trade, this paper is also closely related to Huang and Liu (2004a). They try to reconcile the welfare consequence of unilateral monetary expansion under PCP and LCP. Our analysis differs because we allow for stage-specific productivity shocks and focus on the *optimal monetary policy* under vertical structure of production and trade.

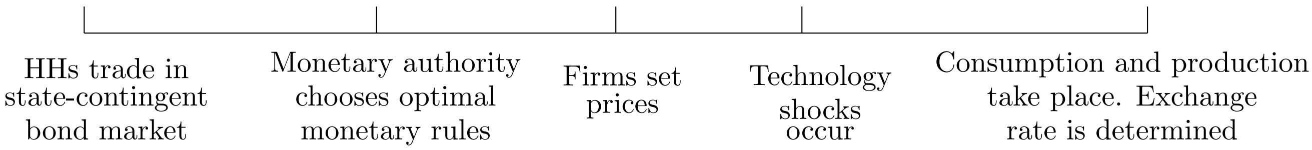
This paper is organized as follows. Section 2 presents the basic model. Section 3 gives the flexible price solution of the model. Section 4 analyzes the optimal monetary policy response to stage-specific productivity shocks and its welfare implication. Section 5 extends the model to include LCP pricing in the model. Section 6 concludes.

2 Basic Model

The world consists of two countries of the same size, denoted as the home country and the foreign country. Each country has one unit of population; households derive utility from aggregated consumption (composed of home finished goods and foreign finished goods), real balance, and leisure. Our assumption about the vertical trade is similar to that in Huang and Liu (2000, 2004b). There are two stages of production in each country: One is the finished goods stage, the other is the intermediate goods stage, and both kinds of goods are tradable. Each stage in each country has a stage-specific productivity shock. Also, there is a continuum of firms indexed on the interval [0,1] in both stages. Each firm produces differentiated goods and therefore has some monopolistic power. The production of finished goods requires a basket of distinct variety of domestic intermediate goods and a basket of distinct variety of imported intermediate goods. The production of the intermediate goods requires only labor. Figure 2 gives the structure of the economy. All firms set prices before the realization of the shocks, and the prices are in the currency of the producer.

For simplicity, we abstract from any dynamics by considering a single-period model with uncertainty[3]. The structure of events within the period is shown in Figure 1.

Figure 1: Timing of Model



First, before the period begins, households can trade in the bond market for a full set of nominal state-contingent bonds. Then monetary authorities choose optimal monetary rules, given the cross-country risk-sharing rule, taking into account the way in which firms set prices, as well as the distribution of stochastic productivity shocks. Following this, firms set prices, given the state-contingent discount factors, the expected demand, and the expected marginal costs. After the realization of stochastic shocks, households work and choose their optimal consumption baskets, production and consumption take place, and the exchange rate is determined.

The detailed structure of the economy in the home country is described below. The foreign country is entirely analogous. From now on, foreign variables and foreign currency prices will be indicated by an asterisk.

2.1 Household

The representative household in the home country maximizes the following expected utility[4]:

$$U = E(\frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{M}{P} - \eta L) \tag{2.1}$$

[3] The results will carry over to an infinite horizon setting without change because we have assumed (a) a full set of nominal state-contingent assets, and (b) ex ante price setting.

[4] The adoption of this utility function will give us a closed-form solution. It is used extensively in the literature; see, for example, Obstfeld and Rogoff (2002) and Devereux and Engel (2003).

where

$$C = 2C_h^{\frac{1}{2}} C_f^{\frac{1}{2}} \qquad\qquad C_h = [\int_0^1 C_h(i)^{\frac{\lambda-1}{\lambda}}]^{\frac{\lambda}{\lambda-1}} \tag{2.2}$$

Here C is the aggregate consumption, C_h is the sub-aggregate consumption of a continuum of home finished goods indexed by $[0, 1]$, C_f is the sub-aggregate consumption of a continuum of imported foreign finished goods. $\frac{M}{P}$ is the real money balances, and L is the home labor supply. $\lambda > 1$, is the elasticity of substitution between differentiated home (foreign) finished goods. ρ is the inverse of the intertemporal elasticity of substitution and is assumed to be greater than 1. From equation (2.2), we can derive the CPI price index and the individual demand for finished goods i in the domestic market and foreign market, respectively[5].

$$P = P_h^{\frac{1}{2}} P_f^{\frac{1}{2}} \tag{2.3}$$

$$C_h(i) = \frac{1}{2}(\frac{P_h(i)}{P_h})^{-\lambda}(\frac{P_h}{P})^{-1}C \tag{2.4}$$

$$C_h^*(i) = \frac{1}{2}(\frac{P_h^*(i)}{P_h^*})^{-\lambda}(\frac{P_h^*}{P^*})^{-1}C \tag{2.5}$$

where $P_h(P_h^*)$ is the price index for home finished goods sold in home (foreign) country; $P(P^*)$ is the home(foreign) CPI price index.

Home and foreign households can trade a full set of state-contingent nominal bonds; thus, the budget constraint of the home households for a particular state of the world z is written as:

$$P(z)C(z) + M(z) + \sum_{\xi \in Z} q(\xi)B(\xi) = W(z)L(z) + \Pi(z) + B(z) + M_0 + T(z) \tag{2.6}$$

That is, the household derives income from the labor income $W(z)L(z)$, the payoff of the state-contingent securities $B(z)$, the profits from their ownership of all home firms $\Pi(z)$, the initial money balance M_0, and the lump-sum transfer from the government $T(z)$. The household chooses how many state-contingent bonds to purchase before the period begins, with $q(\xi)$ and $B(\xi)$ representing the price and holding, respectively, of a security paying off 1 unit of home currency in state $\xi \in Z$, where Z is the set of states. Then the

[5]For details, please see Technical Appendix.

household choose the money holding, consumption, and the labor supply. It is assumed that the government repays any seignorage revenue through the lump transfer, so that $M_0 - M(z) + T(z) = 0$. The specific money supply process will be discussed in later sections.

The trade in state-contingent nominal assets across countries will lead to the following optimal risk-sharing arrangement:

$$\frac{C^{-\rho}}{P} = \Gamma \frac{C^{*-\rho}}{SP^*}, \tag{2.7}$$

where S is the nominal exchange rate, and $P^* = P_h^{*\frac{1}{2}} P_f^{*\frac{1}{2}}$ is the foreign price level. Γ is the state-invariant weight and equals 1[6]. Equation (2.7) implies that one dollar can get the same marginal utility of consumption across countries. Therefore, the real exchange rate is equal to the ratio of marginal utilities of consumption across countries. In addition, the household's optimization problem gives rise to the money demand equation:

$$M = \chi P C^{\rho}, \tag{2.8}$$

and the implicit labor supply function:

$$W = \eta P C^{\rho}. \tag{2.9}$$

Equation (2.8) and (2.9) imply that the nominal wage is proportional to the amount of money in circulation.

2.2 Finished goods stage

There is a continuum of firms indexed by $i \in [0, 1]$ in each country's finished goods stage. Home finished goods firm i produces $Y(i)$ using home and foreign intermediate goods,

[6] Γ represents the ratio of the Lagrange multiplier associated with the home household's budget constraint to the Lagrange multiplier associated with the foreign household's budget constraint. It is also a condition capturing the initial distribution of wealth. Devereux and Engel (2003) show that Γ equals 1 in a symmetric equilibrium.

according to the following production function[7]:

$$Y(i) = 2\theta_F X_h(i)^{\frac{1}{2}} X_f(i)^{\frac{1}{2}} \tag{2.10}$$

where θ_F is the finished goods stage-specific shock in the home country, and $X_h(i)$ ($X_f(i)$) is a basket of distinctive variety of intermediate goods produced in the home (foreign) country.

Cost minimization Each finished goods producer i takes the prices of intermediate goods as given, so the unit cost to produce finished goods i can be derived as:

$$\Lambda = \frac{\tilde{P_h}^{\frac{1}{2}} (S\tilde{P_{fh}^*})^{\frac{1}{2}}}{\theta_F} \tag{2.11}$$

where $\tilde{P_h}$ is the price index of home intermediate goods denominated in home currency, and $\tilde{P_{fh}^*}$ is the price index of foreign intermediate goods, which are sold in the home country, denominated in foreign currency. From the cost minimization problem, we can derive finished goods producer i's demand for the basket of distinct variety of home and foreign intermediate goods:

$$X_h(i) = \frac{1}{2}(\frac{\tilde{P_h}}{\Lambda})^{-1} Y(i) \tag{2.12}$$

$$X_f(i) = \frac{1}{2}(\frac{S\tilde{P}_{fh}^*}{\Lambda})^{-1} Y(i) \tag{2.13}$$

Finished goods price We assume that in each country, the finished goods producer i sets its price in the currency of the producer, thus, the law of one price holds in each individual final goods and purchasing power parity holds in the CPI. From equations (2.4) and (2.5), the total demand for home finished goods i is[8]:

$$Y(i) = C_h(i) + C_h^*(i) = (\frac{P_h(i)}{P_h})^{-\lambda} (\frac{P_h}{P})^{-1} C \tag{2.14}$$

[7]Here, we assume the production of finished goods requires no labor input. This assumption helps us to solve the model analytically. Intuitively, the result of our paper will not change, even if we allow for labor inputs in this stage.

[8]For simplicity, we have used the fact that $C = C^*$ in the total demand function for final goods. Since PPP holds, we could derive $C = C^*$ from the risk-sharing condition (2.7).

Given the demand structure and the unit cost function of finished goods, we can derive the optimal pricing policies for finished goods i from firm i's profit maximization problem:

$$\max_{P_h(i)} E\Pi(i) = E\left[\Omega\,(P_h(i) - \Lambda)\,(\frac{P_h(i)}{P_h})^{-\lambda}(\frac{P_h}{P})^{-1}C\right] \tag{2.15}$$

where $\Omega = \frac{1}{PC^{\rho}}$ is the marginal utility of consumption of the home household, which is used as the stochastic discount factor, as all domestic firms are owned by households. Therefore, we can derive the optimal pricing equation of the home finished goods stage firm

$$P_h(i) = \hat{\lambda}\frac{E[\Lambda C^{1-\rho}]}{E[C^{1-\rho}]} = \hat{\lambda}\frac{E[\frac{S^{\frac{1}{2}}}{\theta_F}C^{1-\rho}]\tilde{P_h}^{\frac{1}{2}}\tilde{P^*_{fh}}^{\frac{1}{2}}}{E[C^{1-\rho}]} \tag{2.16}$$

$$P^*_h(i) = \frac{P_h(i)}{S} \tag{2.17}$$

where $\hat{\lambda} = \frac{\lambda}{\lambda-1}$ is the markup for finished goods pricing. The term $E[\frac{S^{\frac{1}{2}}}{\theta_F}C^{1-\rho}]$ represents a risk premium term arising from the covariance of firm i's profit with marginal utility of consumption, where the fluctuation of the exchange rate S directly affects the firm's pricing decisions. Meanwhile, intermediate goods stage productivity shocks in both the home and foreign countries will affect the price of home finished goods though $\tilde{P}_h$ and $\tilde{P}^*_{fh}$. Imposing symmetry, we can drop out the subscript i. The optimal pricing schedules of the foreign finished goods firms can be derived analogously and are listed in Table 1.

2.3 Intermediate goods stage

It is assumed that there is a continuum of firms indexed by $j \in [0,1]$ in the intermediate goods stage in each country. The home intermediate goods firm j uses the following linear technology, subject to the stage-specific shock θ_I:

$$X_h(j) = \theta_I L(j) \tag{2.18}$$

Demand structure One basket of distinct variety of the home intermediate goods is given by:

$$X_h = [\int_0^1 X_h(j)^{\frac{\phi-1}{\phi}}]^{\frac{\phi}{\phi-1}} \tag{2.19}$$

where ϕ represents the elasticity of substitution between intermediate goods in the home country. The demand for the intermediate goods j in this setting can be derived as:

$$X_h(j) = [\frac{\tilde{P_h}(j)}{\tilde{P_h}}]^{-\phi} X_h \qquad (2.20)$$

where $\tilde{P_h(j)}$ is the price of the intermediate goods j in the home country. Then we may derive home and foreign finished goods producers' total demand for a basket of distinct variety of home intermediate goods, respectively:

$$X_h = \frac{1}{2}(\frac{\tilde{P_h}}{\Lambda})^{-1} \int_0^1 Y(i)di = \frac{1}{2}(\frac{\tilde{P_h}}{\Lambda})^{-1} \int_0^1 [(\frac{P_h(i)}{P_h})^{-\lambda}(\frac{P_h}{P})^{-1}C]di \qquad (2.21)$$

$$X_h^* = \frac{1}{2}(\frac{\frac{\tilde{P_{hf}}}{S}}{\Lambda^*})^{-1} \int_0^1 Y^*(i)di = \frac{1}{2}(\frac{\frac{\tilde{P_{hf}}}{S}}{\Lambda^*})^{-1} \int_0^1 [(\frac{P_f^*(i)}{P_f^*})^{-\lambda}(\frac{P_f^*}{P^*})^{-1}C^*]di \qquad (2.22)$$

Equations (2.21) and (2.22) show that the demand structure of intermediate goods would be affected by aggregate consumption C, the finished goods stage shock θ_F (or θ_F^*) through $\Lambda(\Lambda^*)$, and the nominal exchange rate S.

Intermediate goods prices We assume the prices of intermediate goods are preset in PCP, but we allow for pricing to market. That is, the intermediate goods producer sets two prices: One is for the sales in the domestic market, and the other is for the foreign sales[9]. That is, the law of price may not hold in the intermediate goods stage. The pricing policies for home intermediate goods producer j can be derived from the following profit maximization problem:

$$\begin{aligned}\max_{\tilde{P_h}(j),\tilde{P_{hf}}(j)} E\Pi(j) \quad = \quad & E\{\Omega\{\frac{1}{2}\left(\tilde{P_h}(j) - \frac{W}{\theta_I}\right)\left[\frac{\tilde{P_h}(j)}{\tilde{P_h}}\right]^{-\phi}\left(\frac{\tilde{P_h}}{\Lambda}\right)^{-1}\int_0^1\left[(\frac{P_h(i)}{P_h})^{-\lambda}(\frac{P_h}{P})^{-1}C\right]di \\ & +\frac{1}{2}\left(\tilde{P_{hf}}(j) - \frac{W}{\theta_I}\right)\left[\frac{\tilde{P_{hf}}(j)}{\tilde{P_{hf}}}\right]^{-\phi}\left(\frac{\frac{\tilde{P_{hf}}}{S}}{\Lambda^*}\right)^{-1}\int_0^1\left[(\frac{P_f^*(i)}{P_f^*})^{-\lambda}(\frac{P_f^*}{P^*})^{-1}C^*\right]di\}\}\end{aligned}$$

[9]PCP with pricing to market is used in Devereux, Shi and Xu (2004). With this assumption, we can solve the model analytically.

This yields

$$\tilde{P_h}(j) = \hat{\phi}\frac{E[\frac{W}{\theta_I}\frac{S^{\frac{1}{2}}C^{1-\rho}}{\theta_F}]}{E[\frac{S^{\frac{1}{2}}C^{1-\rho}}{\theta_F}]} \tag{2.23}$$

$$\tilde{P_{hf}}(j) = \hat{\phi}\frac{E[\frac{W}{\theta_I}\frac{S^{-\frac{1}{2}}C^{1-\rho}}{\theta_F^*}]}{E[\frac{S^{-\frac{1}{2}}C^{1-\rho}}{\theta_F^*}]} \tag{2.24}$$

where $\hat{\phi} = \frac{\phi}{\phi-1}$ is the markup for the intermediate goods price. Note that for foreign finished goods producers, the price of home intermediate goods j in terms of the foreign currency is $\frac{\tilde{P_{hf}}(j)}{S}$. If the intermediate goods firms can set price flexibly, then the price simply will be a fixed markup over the unit labor cost. With the sticky price, there is an additional risk premium term arising from the covariance of marginal cost with the term $\frac{S^{\frac{1}{2}}C^{1-\rho}}{\theta_F}$, which represents the demand risk from its buyer–the finished goods producers. Imposing symmetry, we can drop out the subscript j. The optimal pricing schedules of foreign intermediate goods firms are reported in Table 1.

2.4 Stochastic shocks

We assume the final goods stage shock θ_F and intermediate goods stage shock θ_I follow:

$$\theta_F = exp(u), \qquad \theta_I = exp(v) \tag{2.25}$$

where u and v both are mean zero and normally distributed with a variance-covariance matrix[10]

$$\Sigma = \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{pmatrix} \tag{2.26}$$

A similar assumption is made for the foreign productivity shocks. Our setup allows the productivity shock to be specific to a particular stage in a particular country. This nests a special case, where productivity shock are country-specific ($u = v$, $u^* = v^*$).

For simplicity, we assume that the productivity shocks in different stages have the same volatility[11](i.e., $\sigma_u = \sigma_v = \sigma_{u^*} = \sigma_{v^*} = \sigma^2$). We also assume that $\sigma_{uv} = \sigma_{u^*v^*}$. Thus, we have $-\sigma^2 \leq \sigma_{uv} \leq \sigma^2$.

[10]Notice that now we have a continuum of states. Our earlier analysis with a finite number of states extends immediately to this case.

[11]This assumption also helps us to compare the special case with the standard one-stage model.

2.5 Equilibrium

The market clearing conditions of the finished goods market, money market, and labor market are trivial. Note that the intermediate goods market clearing condition in the home country is given by:

$$\theta_I L = X_h + X_h^* = \frac{1}{2}\frac{\Lambda}{\tilde{P}_h}\frac{PC}{P_h} + \frac{1}{2}\frac{\Lambda^*}{\frac{\tilde{P_{hf}}}{S}}\frac{P^*C^*}{P_f^*} \tag{2.27}$$

Thus, given the monetary supply of home and foreign countries[12] M and M^* and the stochastic productivity shocks θ_F, θ_F^*, θ_I, and θ_I^*, the 17 variables C, C^*, P, P^*, P_h, P_f^*, $\tilde{P}_h$, $\tilde{P_{hf}}$, $\tilde{P}_f^*$, $\tilde{P}_{fh}^*$, S, Λ, Λ^*, W, W^*, L, and L^* are determined by 17 equations: risk-sharing condition (2.7); money demand (2.8) and its foreign equivalent; labor supply (2.9) and its foreign equivalent; home finished goods pricing equations (2.16) and its foreign equivalent; intermediate goods pricing equations (2.23) and (2.24), and their foreign equivalents; intermediate goods market clearing condition (2.27) and its foreign analogy; the unit cost function of finished goods (2.11) and its foreign equivalent; and the two CPI price indices. The solution of the model can be easily obtained in closed-form by expressing all endogenous variables as functions of productions shocks (θ_F, θ_F^*, θ_I and θ_I^*) and monetary supply (M and M^*).

3 Flexible Price Solution

First, it is useful to find out the solution of the model in an environment with fully flexible prices[13]. We then may use the terms of trade and the expected utility under the flexible price equilibrium as a benchmark for welfare comparison.

The nominal exchange rate can be derived from the risk-sharing condition (2.7) and money demand equation (2.8):

$$S = \frac{M}{M^*} \tag{3.1}$$

[12]The monetary authority in each country chooses the monetary policy rules to maximize the expected utility of the domestic households. We will discuss the optimal rules in Section 4.

[13]In the economy with two stages of production and trade, there are two sources of distortion associated with monopolistic competition that lower the efficient expected utility level. This inefficiency, however, cannot be eliminated by the monetary policy, so it is not our focus.

From equations (2.7)-(2.9), we have:

$$W = SW^* \tag{3.2}$$

Since the law of one price and PPP holds in CPI level, we have:

$$C = C^* = \left(\frac{M^{\frac{1}{2}} M^{*\frac{1}{2}}}{\chi P_h^{\frac{1}{2}} P_f^{*\frac{1}{2}}}\right)^{\frac{1}{\rho}} \tag{3.3}$$

Putting the home and foreign finished goods price indices together, using the risk-sharing condition, the pricing equation of intermediate goods, and the labor supply function, we may get the solution for the home and foreign consumption in the flexible price equilibrium.

$$C = C^* = (\hat{\lambda}\hat{\phi}\eta)^{-\frac{1}{\rho}} (\theta_F \theta_F^* \theta_I \theta_I^*)^{\frac{1}{2\rho}} \tag{3.4}$$

From the intermediate goods market clearing condition (2.27), we may solve for the home and foreign labor in the flexible price equilibrium.

$$L = L^* = \frac{1}{\hat{\lambda}\hat{\phi}\eta} C^{1-\rho} = (\hat{\lambda}\hat{\phi}\eta)^{-\frac{1}{\rho}} (\theta_F \theta_F^* \theta_I \theta_I^*)^{\frac{1-\rho}{2\rho}} \tag{3.5}$$

Note that both the consumption and the employment are functions of the geometric-weighted average of productivity shocks.

The terms of trade under flexible prices in both stages also can be derived:

$$\frac{SP_f^*}{P_h} = \frac{\theta_F}{\theta_F^*}, \qquad \frac{S\tilde{P}_{hf}^*}{\tilde{P}_h} = \frac{\theta_I}{\theta_I^*} \tag{3.6}$$

To achieve the constrained Pareto efficient allocation, the terms of trade in both the finished goods stage and the intermediate goods stage must be adjusted completely according to the corresponding relative productivity shocks[14].

Following Obstfeld and Rogoff (1998, 2002), it is assumed that the utility derived from real balances is small enough to be neglected. From equation (3.5), the expected utility for the representative consumer in the home country therefore can be measured by the following expression:

$$E\left(\frac{C^{1-\rho}}{1-\rho} - \eta L\right) = \kappa E C^{1-\rho} \tag{3.7}$$

[14] In the flexible price equilibrium, the relative price of finished goods to intermediate goods in each country also will respond completely to the corresponding relative productivity shocks.

where $\kappa = \frac{\lambda\phi-(1-\rho)(\lambda-1)(\phi-1)}{(1-\rho)\lambda\phi} < 0$. Since consumption has the log normal property, we may rewrite $EC^{1-\rho} = \exp{(1-\rho)}(Ec + \frac{1-\rho}{2}\sigma_c^2)$; thus, maximizing the expected utility level is equivalent to maximizing $U_0 = Ec + \frac{1-\rho}{2}\sigma_c^2$. Therefore, we can measure the expected utility level in the flexible price equilibrium as

$$U_0(flex) = \frac{-\ln[\hat{\lambda}\hat{\phi}\eta]}{\rho} + \frac{1-\rho}{8\rho^2}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + 2\sigma_{uv} + 2\sigma_{u^*v^*}] \tag{3.8}$$

In the absence of nominal rigidities, not surprisingly, the equilibrium is independent of the monetary policy. The expected utility level is composed of two parts: a constant function of parameters induced by monopolistic distortions and the impact of stochastic productivity shocks on the expected utility. When $\rho = 1$, the expected utility is affected only by the mean of the log consumption. However, when $\rho > 1$, the risk-averse households care about the uncertainty brought by the stochastic productivity shocks, and the expected utility decreases in the volatilities of the stochastic productivity shocks.

4 Optimal Money Rules

In this section, we study the optimal monetary policy rules in response to stage-specific productivity shocks. The independent monetary authority in each country sets the following monetary rules to maximize the expected utility of the domestic households[15]:

$$m = m_0 + a_1u + a_2u^* + a_3v + a_4v^* \tag{4.1}$$

$$m^* = m_0 + b_1u + b_2u^* + b_3v + b_4v^* \tag{4.2}$$

From now on, let $x = lnX$. The policy parameter vectors $[a_1, a_2, a_3, a_4]$ and $[b_1, b_2, b_3, b_4]$ will be determined by the international monetary Nash game between two independent monetary authorities. It is assumed that they can commit to their monetary rules and that the rules are announced before the firms set their prices.

[15]The log-linear money supply rule we chose here is a general form of policy rule, as consumption, exchange rate and prices are log-linear and the shocks are log normal. Please see Appendix of Devereux and Engle(2003). Thus, the log money supply follows random walk. This property will give us a constant nominal interest rate and shut down the effect of the nominal interest rate on the real balance because the real balance takes log form in the utility function.

As shown in Devereux and Engel (2003), the expected utility level in a stochastic environment is a function of variance and covariance terms of the home and foreign (log) consumption and (log) exchange rate. Thus, once we solve for consumption and exchange rate, we may rewrite the expected utility as functions of the monetary policy parameters. Since prices are set before the realization of the shocks, in log terms, we may write equations (3.1) and (3.3) as

$$s - Es = m - m^* \tag{4.3}$$

$$c - Ec = c^* - Ec^* = \frac{1}{2\rho}(m + m^*) \tag{4.4}$$

where Ex denotes the conditional expectation of the variable x before the period begins. Note that consumption in both countries is determined by the home money supply and the foreign money supply.

We now turn to the derivation of the objective functions of monetary authorities. Using the intermediate goods market clearing condition (2.27), we have:

$$EL = \frac{1}{\hat{\lambda}\hat{\phi}\eta} EC^{1-\rho} \tag{4.5}$$

Thus, the objective function of the monetary authority in the home country is exactly the same as equation (3.7).

From equation (3.7), the expected utility level depends mainly on the mean and the variance of the log consumption, so the key step is to solve for Ec. The detailed derivation is given in the Technical Appendix.

$$\begin{aligned} Ec = & \frac{-ln[\hat{\lambda}\hat{\phi}\eta]}{\rho} - \frac{(2-\rho)}{2}\sigma_c^2 - \frac{1}{4\rho}\sigma_s^2 - \frac{1}{4\rho}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + \sigma_{uv} + \sigma_{u^*v^*}] \\ & + \frac{1}{4\rho}[\sigma_{su} - \sigma_{su^*}] + \frac{1}{4\rho}[\sigma_{sv} - \sigma_{sv^*}] + \frac{1}{2\rho}[\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}] \end{aligned} \tag{4.6}$$

Given the log normal property of consumption and the functional form of the expected utility (3.7), the optimization problem of the monetary authority is equivalent to maximizing $U_0 = Ec + \frac{1-\rho}{2}\sigma_c^2$, which can be derived as

$$\begin{aligned} U_0 = & \frac{-ln[\hat{\lambda}\hat{\phi}\eta]}{\rho} - \frac{1}{2}\sigma_c^2 - \frac{1}{4\rho}\sigma_s^2 - \frac{1}{4\rho}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + \sigma_{uv} + \sigma_{u^*v^*}] \\ & + \frac{1}{4\rho}[\sigma_{su} - \sigma_{su^*}] + \frac{1}{4\rho}[\sigma_{sv} - \sigma_{sv^*}] + \frac{1}{2\rho}[\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}] \end{aligned} \tag{4.7}$$

We may rewrite the expected utility function as a function of policy parameter vectors $\{a\}$ and $\{b\}$. The detailed derivation of the variance and covariance terms of c and s, in terms of the monetary policy parameters is given in the Technical Appendix. For the home households, consumption variance and exchange rate variance reduce utility, while the covariances of consumption and productivity shocks, a positive covariance of the exchange rate and home productivity shocks, or a negative covariance with foreign productivity shocks increases utility. The optimal monetary rule will be a trade-off between these costs and benefits and the effects of monetary policies on both the consumption and the exchange rate will be considered.

A Nash equilibrium of the international monetary game between the home and foreign countries is characterized by the following conditions:

$$(P1) \qquad \begin{aligned} &\max_a U_0(a, b^N) \\ &\max_b U_0^*(a^N, b) \end{aligned}$$

The objective function U_0^* is identical to U_0 because home and foreign countries have identical consumption and employment in equilibrium. This means that the solution to (P1) is identical to the solution of a *cooperative* monetary policy game, where monetary rules are chosen to maximize the joint welfare. In other words, there are no gains to monetary policy coordination.

Solving the international monetary game, we can have the following propositions.

Proposition 1 *The solution to the monetary Nash game (P1) is*

$$a_1^n = \frac{3}{4}, \quad a_2^n = \frac{1}{4}, \quad a_3^n = \frac{3}{4}, \quad a_4^n = \frac{1}{4}$$

$$b_1^n = \frac{1}{4}, \quad b_2^n = \frac{3}{4}, \quad b_3^n = \frac{1}{4}, \quad b_4^n = \frac{3}{4}$$

Proof: See Technical Appendix.

Therefore, the solution to the Nash game in our model implies that the optimal monetary policy requires the home monetary authority to respond positively to both home and foreign productivity shocks, though the response of the home monetary authority to domestic productivity shocks exceeds that of the foreign monetary authority. This result is quite different from Obstfeld and Rogoff (2002) and Devereux and Engel (2003), where the optimal monetary policy requires the home monetary policy to respond only to the home productivity shock with PCP pricing and full exchange rate pass-through[16]. That is, if there are two country-specific shocks θ and θ^*, the policy parameter vector would be $[a_1 = 1,\ a_2 = 0]$ and $[b_1 = 0,\ b_2 = 1]$.

Why does foreign monetary authority respond positively to home productivity shocks in our model? This is because home productivity shocks have a trans-border spillover effect on the foreign country, which is induced by the vertical structure of production and trade. To understand this effect, we first explain what an optimal monetary policy requires in the case without vertical trade. Suppose there is a productivity shock in home country, as prices are sticky, consumption and employment do not respond to this shock as they would in a flexible price equilibrium. So the optimal monetary response to this shock must be expansionary, so as to shift demand up to meet the increased supply. Meanwhile, optimal monetary policy also should change relative prices to increase the relative demand for home goods. For this to happen, the nominal exchange rate should depreciate to ensure that the world demand is shifted towards home-produced goods. Therefore, the optimal monetary response to home productivity shock should have both the "level" effect and the "expenditure-switching" effect. With only one stage of trade, $[a_1 = 1,\ a_2 = 0]$ and $[b_1 = 0,\ b_2 = 1]$ will achieve both the desired aggregate demand increase and the desired terms of trade adjustment in response to country-specific productivity shocks.

When there is more than one stage of production and trade, given the PCP pricing, the exchange rate changes will affect terms of trade in multiple stages. Hence, the productivity shocks in one stage and one country can translate into relative demands shocks in other stages and other countries. For instance, if a positive stage-specific productivity shock

[16] In their model, monetary authorities react to foreign productivity shocks under PCP setting only when the elasticity of money demand is not equal to 1, while in our model this elasticity is 1. Moreover, the mechanism leading to the response to foreign shocks works through changes in the nominal interest rate, which is completely different from that in our model. For example, if elasticity of demand is greater than 1, the foreign productivity shock causes a rise in the nominal interest rate, which will generate excess supply of real balance. To eliminate the effect of this on consumption, the home country money supply must be reduced.

occurs in the home finished goods stage, the supply of home finished goods will increase. So the world money supply should be increased to shift up the demand for home finished goods. Meanwhile, the expenditure-switching effect requires the exchange rate to depreciate. Nevertheless, the exchange rate depreciation not only increases the relative demand for home finished goods but also makes home intermediate goods relatively cheaper, which is equivalent to a negative relative demand shock to the foreign country. Therefore, the foreign monetary authority should respond positively to this home finished goods shock so as to eliminate these inefficient terms of trade changes in the intermediate goods stage[17]. Similarly, both the home and foreign country should respond to productivity shocks in the intermediate stage. That is, any stage-specific shock in one country would affect both countries like a "world shock," so both countries should respond positively to this shock.

Intuitively, the expenditure-switching effect should dominate the indirect positive trans-border spillover effect. To achieve this, the home country monetary reaction to a home productivity shock is positive and exceeds that of the foreign country.

The existence of the tran-border spillover effect naturally leads to a question about gains from international monetary coordination in a model like this. As noted above, for analytical tractability, we assume international risk-sharing, unitary elasticity between home and foreign goods, and PCP in the finished goods stages. These assumptions imply that there is no policy coordination gain in the current specification of the model. Nevertheless, if these assumptions are relaxed, there might exist policy coordination gain[18]. Also, Obstfeld and Rogoff (2002), Benigno and Benigno (2003), and Tchakarov (2004) show that there might exist gains from policy coordination in a model with both wage and price rigidities, as multiple stickiness provides a scope for policy coordination. The same logic can be applied to the model with vertical trade and production. Moreover,

[17]The change in terms of trade in the intermediate goods stage induced by optimal response to finished goods stage shock is not efficient, as there are no shocks in this stage.

[18]In the literature, the sources of coordination gains can be related to the degree of exchange rate pass-through (Betts and Devereux, 2000; Corsetti and Pesenti, 2005); values of elasticity of intertemporal substitution and elasticity of substitution between home and foreign goods (Obstfeld and Rogoff, 2002; Benigno and Benigno, 2003; Pappa 2004; Sutherland 2002; Tchakarov 2004); incomplete financial market (Benigno 2001; Sutherland 2004); imperfect correlation of productivity shocks in traded and non-traded sectors (Canzoneri, Cumby and Diba, 2004); and asymmetric production structure (Liu and Pappa, 2005).

the existence of the trans-border spillover effect implies another kind of terms of trade spillover or externality effect that might induce policy coordination gain[19].

From the rules given in Proposition 1, we may establish the following proposition:

Proposition 2 *The optimal monetary policy cannot replicate the flexible price equilibrium, and the flexible exchange rate cannot deliver the economy to the efficient level, unless the productivity shocks in the two stages of production are perfectly correlated.*

Proof: See Appendix A.

When there is only one stage of trade, the conventional wisdom regarding the welfare implication of the flexible exchange rate is that it can bring the economy around the obstacle of nominal rigidities, if the prices are preset in PCP or there is complete exchange rate pass-through. That is, the optimal monetary policy can replicate the flexible price equilibrium. This conclusion, however, does not hold when there is more than one stage of production and trade. The intuition is straightforward. If there is only one stage of production and trade, the flexible exchange rate can adjust the terms of trade to the constrained Pareto efficient level – the level under the flexible prices equilibrium, so the world resource can be allocated efficiently. However, in a world with vertical structure of production and trade, there are two terms of trade between home and foreign countries. Thus, the flexible exchange rate cannot adjust the relative prices of *both* finished and intermediate goods to the efficient level simultaneously, unless the productivity shocks in the two stages are perfectly correlated.

For instance, under the PCP assumption, the terms of trade in the finished goods stage is $\frac{SP_f^*}{P_h}$. Because P_h and P_f^* are both predetermined, the terms of trade will be proportional to the exchange rate changes $\frac{(\theta_F\theta_I)^{\frac{1}{2}}}{(\theta_F^*\theta_I^*)^{\frac{1}{2}}}$, which is different from the terms of trade under the flexible price equilibrium $\frac{\theta_F}{\theta_F^*}$. Similarly, the terms of trade in the intermediate goods stage is not equivalent to its flexible price level either.

[19]One advantage of the current model is that all the results can be derived in closed-form, but it is not an appropriate framework for the discussion of policy coordination. A more general model setting and some numerical exercises will be needed. For reasons of scope, we feel that an analysis of this question could not be pursued in the current paper. In our view, this issue is important enough to deserve investigation in a separate paper.

A similar result in the closed-economy literature is shown by Erceg, Henderson and Levin (2000). They find that when prices and wages both are sticky, the allocation with flexible prices and wages cannot be restored by the optimal monetary policy. That is, if there exist multiple sources of stickiness, one policy instrument cannot deal with all nominal rigidities efficiently. Huang and Liu (2004c) establish an analogous result in a closed economy model with two stages of production.

Here, we emphasize the effect of multiple nominal rigidities on optimal monetary policy in the open economy. Friedman (1953) and later Mundell (1961) argue that the flexible exchange rate can act as an efficient mechanism for dealing with country-specific shocks when the adjustment of domestic price levels is sluggish. But Proposition 2 shows that this argument will not hold when there is vertical chain of production and trade. It implies that the function of the flexible exchange rate as a mechanism to adjust relative prices is limited in the presence of multiple stickiness caused by vertical trade. In the trade literature, vertical specialization is usually considered as welfare-improving. Feenstra (1998) argues that it brings efficiency gains that amount to an outward shift in the production frontier for final goods in each country. Meanwhile, it also moves factor prices towards greater equality globally. Nevertheless, we show that vertical trade might bring a welfare cost in the macroeconomic level because of the multiple nominal rigidities.

Given the optimal monetary rules, we can compare the maximized expected utility level under the sticky price equilibrium with the expected utility level under the flexible price equilibrium:

$$U_0(flex) - U_0(sticky) = \frac{1}{16\rho}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 - 2\sigma_{uv} - 2\sigma_{u^*v^*}] \geq 0 \tag{4.8}$$

This welfare difference can be used to measure the cost of multiple stickiness coming from vertical production and trade. Equation (4.8) also implies a higher correlation between finished goods shock and intermediated goods shock in each country increases the welfare under optimal monetary policy. When the shocks in two different stages of one country are perfectly correlated, the optimal monetary rules can replicate the flexible price equilibrium[20], but the response of monetary authorities to shocks still is different from that in the standard open economy literature.

[20]The welfare difference between the flexible price equilibrium and the sticky price equilibrium is zero.

Finally, we can have the following proposition:

Proposition 3 *Given the pricing structure and the variance covariance matrix of productivity shocks (2.26) under the flexible exchange rate regime, the exchange rate is more stable when there is vertical production and trade.*

Proof: See Appendix A.

The intuition of Proposition 3 is as follows; With vertical structure of production and trade, the production of the world import and export goods is more diversified, but each country is more integrated with other countries. Therefore, even if terms of trade can be adjusted, there is no need for large exchange rate changes to adjust the world economy according to the relative shocks.

In the special case where shocks are country-specific(i.e., $u = v$, $u^* = v^*$), then obviously Proposition 1 still holds; countries still should respond positively to foreign shocks. As to Proposition 2, now the optimal monetary policy can replicate the flexible price equilibrium, as $u = v(u^* = v^*)$ implies perfect correlation. Finally, the exchange rate volatility under optimal monetary policy in this special case is just equal to that of the one-stage model[21].

Thus, our findings suggest that the introduction of vertical structure and trade does affect the international transmission mechanism of productivity shocks and the optimal monetary policy design in an open economy. With vertical trade, the foreign productivity shock has a trans-border spillover effect on the home country and thus, the optimal monetary policy should respond to foreign shocks. Moreover, we find that even under PCP pricing, the flexible exchange rate no longer could replicate the flexible price equilibrium. Therefore, the implication of flexible exchange rate for the replication of flexible price equilibrium depends on whether shocks are country- or stage-specific. If the policy makers are not able to observe the nature of shocks, the misconduct of monetary policy might lead to inefficient outcome and higher exchange rate volatility.

[21] Intuitively, this is because there is only one source of real shock in both models.

5 Extension: Vertical Trade with LCP

From the above section, several interesting questions can be raised. First, since the trans-border spillover effect works through the relative demand of goods, does it depend on the pricing behavior of firms? Second, since exchange rate could not adjust terms of trade in both stages to the efficient level simultaneously, will vertical trade structure change the value of exchange rate flexibility under PCP and LCP? For instance, if there exists LCP in one stage of the production and hence, the exchange rate can focus on the terms of trade in the other stage and adjust it to the efficient level, will optimal monetary policy deliver a higher welfare than the benchmark model in Section 2? Huang and Liu (2004) show that vertical chain of production and trade can magnify the efficiency-improvement effect of a unilateral monetary expansion, while dampening its terms of trade effect. Therefore, when there is vertical trade, the welfare consequence of unilateral monetary shock under PCP can be reconciled. Here, we are interested in finding out the welfare implication of PCP and LCP under *optimal* monetary policy. To answer these questions, we extend the model to allow for LCP in the intermediate goods stage[22].

5.1 The Pricing Decisions

The problem of the firms in the finished goods stage is exactly the same as before.

$$P_h(i) = \hat{\lambda}\frac{E[\Lambda C^{1-\rho}]}{E[C^{1-\rho}]} \qquad P_h^*(i) = \frac{P_h(i)}{S} \tag{5.1}$$

where Λ, the unit cost for home finished goods firms is now different from the pure PCP case

$$\Lambda = \frac{\tilde{P_h}^{\frac{1}{2}}(\tilde{P_{fh}})^{\frac{1}{2}}}{\theta_F} \tag{5.2}$$

[22]Devereux and Engel (2005) show that exchange rate policy acts as a trade-off between the desire to smooth fluctuations in the real exchange rate for risk-sharing, and the need to allow flexibility in the nominal exchange rate, so as to facilitate terms of trade adjustment. If export prices are set in LCP in the finished goods stage, PPP will not hold, and the exchange rate policy will reflect the consideration of risk-sharing. So we assume LCP only in intermediate stage so as to focus on the role of exchange rate changes on terms of trade adjustment. Also, the model will lose its analytical tractability if we allow for LCP in the finished goods stage.

where $\tilde{P_h}$ is the price index of home intermediate goods, and $\tilde{P_{fh}}$ is the price index of foreign intermediate goods sold in the home country, denominated in home currency, as we assume LCP in the intermediate goods stage. Similarly, the unit cost for foreign finished goods firms is now given by

$$\Lambda^* = \frac{\tilde{P_f^*}^{\frac{1}{2}} (\tilde{P_{hf}^*})^{\frac{1}{2}}}{\theta_F^*} \tag{5.3}$$

where $\tilde{P_f^*}$ is the price index of foreign intermediate goods in foreign, and $\tilde{P_{hf}^*}$ is the price index of home intermediate goods sold in the foreign country, denominated in foreign currency. The home intermediate goods firm j now faces the following demand structure:

$$X_h(j) = [\frac{\tilde{P}_h(j)}{\tilde{P}_h}]^{-\phi} \frac{1}{2} (\frac{\tilde{P}_h}{\Lambda})^{-1} \int_0^1 [(\frac{P_h(i)}{P_h})^{-\lambda} (\frac{P_h}{P})^{-1} C] di \tag{5.4}$$

$$X_h^*(j) = [\frac{\tilde{P_{hf}^*}(j)}{\tilde{P_{hf}^*}}]^{-\phi} \frac{1}{2} (\frac{\tilde{P_{hf}^*}}{\Lambda^*})^{-1} \int_0^1 [(\frac{P_f^*(i)}{P_f^*})^{-\lambda} (\frac{P_f^*}{P^*})^{-1} C^*] di \tag{5.5}$$

So the profit maximization problem of firm j is

$$\max_{\tilde{P}_h(j),\, \tilde{P_{hf}^*}(j)} E\Pi(j) = E\left\{ \Omega \left[\left(\tilde{P}_h(j) - \frac{W}{\theta_I} \right) X_h(j) + \left(S\tilde{P_{hf}^*}(j) - \frac{W}{\theta_I} \right) X_h^*(j) \right] \right\} \tag{5.6}$$

This yields

$$\tilde{P}_h = \frac{[E(\frac{W}{\theta_I} \frac{C^{1-\rho}}{\theta_F})]}{E(\frac{C^{1-\rho}}{\theta_F})} \qquad \tilde{P_{hf}^*} = \frac{[E(\frac{W}{\theta_I} \frac{S^{-1}C^{1-\rho}}{\theta_F^*})]}{E(\frac{C^{1-\rho}}{\theta_F^*})} \tag{5.7}$$

The prices set by foreign firms under this pricing structure are listed in Table 1.

The next step is to derive the mean of the log consumption in terms of variance and covariance of the log consumption, the log exchange rate, and the productivity shocks.

$$\begin{aligned} Ec &= \frac{-ln[\hat{\lambda}\hat{\phi}\eta^2]}{2\rho} - \frac{(2-\rho)}{2}\sigma_c^2 - \frac{1}{8\rho}\sigma_s^2 - \frac{1}{4\rho}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + \sigma_{uv} + \sigma_{u^*v^*}] \\ &\quad + \frac{1}{4\rho}(\sigma_{su} - \sigma_{su^*}) + \frac{1}{2\rho}[\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}] \end{aligned} \tag{5.8}$$

5.2 Optimal Monetary Policy

Because we assume PCP in finished goods stage, equation (2.7) implies that $C = C^*$ still holds. Using the intermediate goods market clearing condition $\theta_I L = X_h + X_h^* =$

$\frac{1}{2}\frac{\Lambda}{\tilde{P}_h}\frac{PC}{P_h} + \frac{1}{2}\frac{\Lambda^*}{\tilde{P}^*_{hf}}\frac{P^*C^*}{P^*_f}$, we have $EL = \frac{1}{\hat{\lambda}\hat{\phi}\eta}EC^{1-\rho}$. Then using equation (5.8), we could derive the expression of expected utility

$$
\begin{aligned}
EU &= (\frac{1}{1-\rho} - \frac{1}{\hat{\lambda}\hat{\phi}}) \exp\{(1-\rho)\{\frac{-ln[\hat{\lambda}\hat{\phi}\eta^2]}{2\rho} - \frac{1}{4\rho}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + \sigma_{uv} + \sigma_{u^*v^*}] \\
&\quad -\frac{1}{2}\sigma_c^2 - \frac{1}{8\rho}\sigma_s^2 + \frac{1}{4\rho}(\sigma_{su} - \sigma_{su^*}) + \frac{1}{2\rho}[\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}]\}\}, \qquad (5.9)
\end{aligned}
$$

which is a function of policy vectors $\{a\}$ and $\{b\}$. Since $C = C^*$ and $EL = EL^* = \frac{1}{\hat{\lambda}\hat{\phi}\eta}EC^{1-\rho}$, the objective function of home and foreign are identical. Therefore, similar to the benchmark model, there is no policy coordination gain in this economy either.

The Nash equilibrium of international monetary policy game is then given by

$$a = [1, 0, \frac{1}{2}, \frac{1}{2}], \qquad b = [0, 1, \frac{1}{2}, \frac{1}{2}] \qquad (5.10)$$

Therefore, we can see that now the domestic monetary authority does not respond to the foreign finished goods stage productivity shock. In other words, there is no trans-border spillover effect in the extended model. This is because the LCP in the intermediate stage eliminates the relative demand changes due to exchange rate adjustments. When θ_F increases, supply of home finished goods increases. As prices are sticky, both the "level" effect and the "expenditure-switching" effect imply increases in home money supply. Because now exchange rate changes could not affect the relative price in the intermediate stage, the optimal response is to adjust terms of trade in the finished goods stage to its efficient level. Thus, home monetary policy should respond fully to domestic shock only, just as in the one-stage model.

Although the optimal monetary policy response to foreign intermediate stage shocks is positive, this is not due to the trans-border spillover effect. If θ_I increases, supply of home intermediate goods increases. So the demand for home intermediate goods should increase as well, which can be achieved only by increases in aggregate demand or relative demand. LCP in intermediate goods implies the "expenditure-switching effect" does not exist in this model. So both home and foreign countries should increase the real balance, so as to increase the aggregate demand.

Therefore, the trans-border spillover effect depends not only on vertical structure of production and trade but also on the currencies of pricing setting in production stages. It works through the relative price adjustment caused by exchange rate changes, so currency of pricing matters.

5.3 Welfare Comparison

Now we compare the welfare implication of PCP and LCP under optimal monetary policy in a vertical trade model. Given (5.10), the home (foreign) country representative household's welfare can be measured by[23]:

$$U_0(pcp, lcp) = -\frac{\ln[(\hat{\lambda}\hat{\phi}\eta)^2]}{2\rho} + \frac{1-\rho}{8\rho^2}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + 2\sigma_{uv} + 2\sigma_{u^*v^*}] - \frac{1}{8\rho}(\sigma_v^2 + \sigma_{v^*}^2) \quad (5.11)$$

With pure PCP structure, the welfare under the optimal monetary policy is

$$\begin{aligned} U_0(pcp, pcp) \quad = \quad & -\frac{\ln[(\hat{\lambda}\hat{\phi}\eta)^2]}{2\rho} + (\frac{1}{8\rho^2} + \frac{1}{16\rho})[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + 2\sigma_{uv} + 2\sigma_{u^*v^*}] \\ & -\frac{1}{4\rho}[\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + \sigma_{uv} + \sigma_{u^*v^*}] \end{aligned} \quad (5.12)$$

So the welfare difference is

$$U_0(pcp, lcp) - U_0(pcp, pcp) = \frac{1}{16\rho}[\sigma_u^2 + \sigma_{u^*}^2 - \sigma_v^2 - \sigma_{v^*}^2 - 2\sigma_{uv} - 2\sigma_{u^*v^*}] \quad (5.13)$$

Thus, given our assumption of the variance-covariance matrix of shocks $\Sigma = \begin{pmatrix} \sigma^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma^2 \end{pmatrix}$, we have the following proposition:

Proposition 4 *If* $\sigma_{uv} = 0$*, then* $U_0(pcp, lcp) = U_0(pcp, pcp)$*.If* $\sigma_{uv} > 0$*, then* $U_0(pcp, lcp) < U_0(pcp, pcp)$*. If* $\sigma_{uv} < 0$*, then* $U_0(pcp, lcp) > U_0(pcp, pcp)$*.*

Proof: Straightforward from equation (5.13).

Therefore, the optimal monetary policy in the case with LCP in the intermediate goods stage can deliver a higher welfare than that in the case with PCP in both stages, when the technology shocks in the finished goods stage and intermediate goods stage are negatively correlated. However, in a one-stage model like Devereux and Engel (2003), the welfare given by optimal monetary policy under PCP is always higher than that under LCP. The

[23] Note that $EU^x = (\frac{1}{1-\rho} - \frac{1}{\hat{\lambda}\hat{\phi}}) \exp[(1-\rho)U_0^x]$. So maximizing EU^x is equivalent to maximizing U_0^x.

intuition is straightforward. With PCP, the optimal monetary policy replicates the flexible price equilibrium. With LCP, exchange rate changes could not adjust the terms of trade to allocate the resources in response to country-specific shocks efficiently, so the welfare is always lower.

In the model with vertical production and trade, however, LCP in the intermediate stage can deliver a higher welfare than PCP in both stages. Intuitively, this is because the value of exchange rate flexibility in the pure PCP model depends on the correlation of productivity shocks in two stages, while the value of exchange rate flexibility in the case with LCP pricing depends only on the variance of productivity shocks in the finished goods.

Following Obstfeld and Rogoff (2002), we could use the welfare difference between the "optimal fixed regime"[24] and the "optimal float regime" to measure the value of exchange rate flexibility under two different pricing structure assumptions. It is easy to show that the optimal fixed regime in both the benchmark model and the extended model is:

$$a_{fix} = b_{fix} = [\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}] \tag{5.14}$$

So the value of exchange rate flexibility under two pricing assumptions is given by:

$$U_0^{flex}(pcp, lcp) - U_0^{fix}(pcp, lcp) = \frac{1}{16\rho}(\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 + 2\sigma_{uv} + 2\sigma_{u^*v^*}) \tag{5.15}$$

$$U_0^{flex}(pcp, lcp) - U_0^{fix}(pcp, lcp) = \frac{1}{8\rho}(\sigma_u^2 + \sigma_{u^*}^2) \tag{5.16}$$

Obviously, the value of exchange rate flexibility in the case with PCP will be lower when stage-specific shocks are negatively correlated, while in the case with LCP it depends only on the variance of productivity shocks in the finished goods.

The intuition is as follows: In the benchmark model, since prices are fixed ex ante, given the optimal monetary policy, the terms of trade in both stages are proportional to the exchange rate changes $\frac{(\theta_F\theta_I)^{\frac{1}{2}}}{(\theta_F^*\theta_I^*)^{\frac{1}{2}}}$. If the shocks are positively correlated, then they are closer to the terms of trade under the flexible price equilibrium $\frac{\theta_F}{\theta_F^*}$ and $\frac{\theta_I}{\theta_I^*}$. When

[24] To find the optimal fixed regime, we should assume that the home and foreign monetary authorities respond identically to the productivity shocks. That is, $a = b$.

productivity shocks in two stages are negative correlated, the terms of trade in both stages are further away from their flexible price counterparts. While in the extended case, optimal monetary policy can bring the terms of trade in the finished goods stage to the flexible price level but has no effect on the terms of trade in intermediate stages. So the correlation of productivity shocks will not affect value of exchange rate flexibility in the model with LCP. When the correlation of two stage-specific productivity shocks is negative, the value of exchange rate flexibility is actually higher in the LCP case than in the PCP case. So the optimal monetary policy can deliver a higher welfare for the whole economy. This result illustrates the impact of vertical trade on the value of exchange rate flexibility under different pricing structures.

6 Conclusion

This paper examines the optimal monetary policy in a world with vertical production and trade by introducing two stages of production and trade into the standard utility-based open economy macroeconomic model.

We find that when both finished goods and intermediate goods are tradable, there might exist a trade-induced trans-border spillover effect of productivity shocks, which changes the way monetary authorities respond to other countries' productivity shocks. We also find that the flexible exchange rate cannot bring the economy back to the efficient level, even under the PCP pricing or the complete exchange rate pass-through case.

In the extension of the model, it is found that the trans-border spillover effect depends not only on the vertical production and trade but also on the currencies of price setting. It only exists when terms of trade in multiple stages can be adjusted by the exchange rate. Finally, we show that vertical structure of production and trade will change the value of exchange rate flexibility under PCP and LCP.

Our findings suggest that the changes in the trade pattern in the global economy over the last thirty years might affect the international optimal monetary policy rules and values of exchange rate flexibility. So the monetary policy makers should take into account the impact of changes in the trade pattern when making decisions.

In the subsequent research, with the help of some numerical methods, more stages of production and trade can be considered, and the optimal monetary policy in a more integrated global economy can be explored. Another important future research direction is to explore gains from international monetary policy coordination in a model with vertical production and trade. As explained in Section 4, the existence of the trans-border spillover effect and the fact that the flexible exchange rate could not replicate the flexible price equilibrium in such a model both imply that there is scope for policy coordination gain. We also can explore the role of asymmetries in the vertical production and trade structure across countries in generating gains from policy coordination.

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Appendix

A Proofs of Propositions

A.1 Proof of Proposition 2

First note that when shocks in two stages are perfectly correlated, $\theta_F(\theta_F^*)$ is proportional to $\theta_I(\theta_I^*)$. Substituting the Nash equilibrium in Proposition 1 to equations (C.1)-(C.6) in the Technical Appendix, then using equation (4.6) we could get

$$Ec = \frac{-ln[\hat{\lambda}\hat{\phi}\eta]}{\rho} - \frac{1}{16\rho}(\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 - 2\sigma_{uv} - 2\sigma_{u^*v^*}) \tag{A.1}$$

Using this solution, and the optimal monetary rules, equation (4.4) gives us

$$c = \frac{-ln[\hat{\lambda}\hat{\phi}\eta]}{\rho} - \frac{1}{16\rho}(\sigma_u^2 + \sigma_{u^*}^2 + \sigma_v^2 + \sigma_{v^*}^2 - 2\sigma_{uv} - 2\sigma_{u^*v^*}) + \frac{1}{2\rho}(u + u^* + v + v^*) \tag{A.2}$$

Since we assume $\sigma_u^2 = \sigma_{u^*}^2 = \sigma_v^2 = \sigma_{v^*}^2 = \sigma^2$, from the above equation, we can see that if shocks are perfectly correlated, $\sigma_{uv} = \sigma_{u^*v^*} = \sigma^2$. C is then identical to the flexible price solution in equation (3.4).

From equation (3.1) and the optimal monetary rules, we have $S = \frac{(\theta_F\theta_I)^{\frac{1}{2}}}{(\theta_F^*\theta_I^*)^{\frac{1}{2}}}$. Since $M(M^*)$, $W(W^*)$, $C(C^*)$, and S can all be expressed in terms of productivity shocks, substituting them into the pricing equations for P_h, P_f^*, $\tilde{P}_h$ and $\tilde{P}_{hf}^*$, we can have $P_h = P_f^*$ and $\tilde{P}_h = \tilde{P}_{hf}^*$ when shocks are perfectly correlated. So the terms of trade in finished and intermediate stages are given by $\frac{SP_f^*}{P_h} = S = \frac{\theta_F}{\theta_F^*}$ and $\frac{S\tilde{P}_{hf}^*}{\tilde{P}_h} = S = \frac{\theta_I}{\theta_I^*}$, respectively, which are equal to their flexible price equilibrium counterparts given by equation (3.6).

The labor market equilibrium condition (2.27) in both the flexible price model and the sticky price case can be rewritten as

$$L = \frac{1}{2}\left(\frac{S\tilde{P}_{hf}^*}{\tilde{P}_h}\right)^{\frac{1}{2}} \frac{1}{\theta_F}\left(\frac{SP_f^*}{P_h}\right)^{\frac{1}{2}} C + \frac{1}{2}\left(\frac{S\tilde{P}_f^*}{\tilde{P}_{hf}}\right)^{\frac{1}{2}} \frac{1}{\theta_F^*}\left(\frac{P_h}{P_f^* S}\right)^{\frac{1}{2}} C^* \tag{A.3}$$

Since only when shocks are perfectly correlated, both $C(C^*)$ and terms of trade in two stages in the benchmark model are identical to those in the flexible price model, so will be L.

A.2 Proof of Proposition 3

From the optimal monetary policy rules, we may solve for the log exchange rate:

$$s = \frac{1}{2}u - \frac{1}{2}u^* + \frac{1}{2}v - \frac{1}{2}v^* \tag{A.4}$$

Given the assumption that the shocks in one country have the variance-covariance matrix $\Sigma = \begin{pmatrix} \sigma^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma^2 \end{pmatrix}$, it follows that the variance of optimal exchange rate is $\sigma^2 + \sigma_{uv}$. When there is no correlation between the finished goods shock and the intermediate goods shock in one country ($\sigma_{uv} = 0$), the exchange rate volatility is just σ^2. However, when there is only one stage of production and trade and the price is set under PCP pricing, the optimal policy parameter vectors are $a = [1, 0]$ and $b = [0, 1]$. This implies that the variance of optimal exchange rate is $2\sigma^2$ if the volatilities of both home and foreign country-specific shocks are equal to σ^2 as well. Even if $\sigma_{uv} > 0$, the exchange rate volatility is still lower than that under the economy without vertical structure of production and trade. Therefore, our findings suggest that the floating exchange rate regime is more stable in a world with vertical production and trade.

Table 1: **The Optimal Pricing Policies for Foreign Firms**[a]

PCP		PCP and LCP	
P_f^*	$\frac{\lambda}{\lambda-1}\frac{E[\frac{S^{-\frac{1}{2}}}{\theta_F^*}C^{1-\rho}]\tilde{P_f^*}^{\frac{1}{2}}\tilde{P_{hf}}^{\frac{1}{2}}}{E[C^{1-\rho}]}$	P_f^*	$\frac{\lambda}{\lambda-1}\frac{E[\frac{1}{\theta_F^*}C^{1-\rho}]\tilde{P_f^*}^{\frac{1}{2}}\tilde{P_{hf}^*}^{\frac{1}{2}}}{E[C^{1-\rho}]}$
P_f	P_f^*S	P_f	P_f^*S
$\tilde{P_f}^*$	$\frac{\phi}{\phi-1}\frac{E[\frac{W^*}{\theta_I^*}\frac{S^{-\frac{1}{2}}C^{1-\rho}}{\theta_F^*}]}{E[\frac{S^{-\frac{1}{2}}C^{1-\rho}}{\theta_F^*}]}$	$\tilde{P_f}^*$	$\frac{[E(\frac{W^*}{\theta_I^*}\frac{C^{1-\rho}}{\theta_F^*})]}{E(\frac{C^{1-\rho}}{\theta_F^*})}$
$\tilde{P_{fh}}^*$	$\frac{\phi}{\phi-1}\frac{E[\frac{W^*}{\theta_I^*}\frac{S^{\frac{1}{2}}C^{1-\rho}}{\theta_F}]}{E[\frac{S^{\frac{1}{2}}C^{1-\rho}}{\theta_F}]}$	$\tilde{P_{fh}}$	$\frac{[E(\frac{W^*}{\theta_I^*}\frac{SC^{1-\rho}}{\theta_F})]}{E(\frac{C^{1-\rho}}{\theta_F})}$

[a] The prices with asterisk are in term of foreign currency.

Figure 2: **The Structure of the Economy**