Entry Cost, Tobin Tax, and Noise Trading in the Foreign Exchange Market

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Abstract

Noise trading in the foreign exchange market can cause excess exchange rate volatility, which might reduce gains from international trade and lower welfare. To eliminate the noise in the market, two types of regulatory policies have been proposed: increasing the entry cost to traders and imposing a 'Tobin tax' type of transaction tax on foreign exchange trading. In practice, however, it is difficult for authorities to identify noise traders. This implies that both noise traders and informed traders will be affected by these policies. In this paper, we endogenize the entry decisions of both types of traders and show that these policies may be ineffective in reducing exchange rate volatility, or even have an adverse effect and increase exchange rate volatility. In equilibrium, exchange rate volatility is determined by the composition of traders. Increasing entry costs will discourage the entry of all traders, but it may not change the relative ratio of traders, or it may affect informed traders disproportionately more, which increases the relative ratio of noise traders and exchange rate volatility. Furthermore, we find that a Tobin tax may also increase exchange rate volatility when both types of traders' endogenous entry decisions are considered. This is because the interaction between a Tobin tax and entry costs may lead to an increase in the relative noise component and exchange rate volatility.

JEL classification: F3, G1

Keywords: Noise trader; Informed trader; Exchange rate volatility; Entry cost; Tobin Tax

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1 Introduction

It is widely acknowledged that noise trading can cause excessive volatility in financial markets, especially in the foreign exchange market. For example, Shleifer and Summers (1990) and Shleifer and Vishny (1997) argue that noise trading introduces risks that inhibit arbitrageurs and prevent prices from converging to fundamental asset values. Jeanne and Rose (2002) show that exchange rate volatility has two components: fundamentals and noise. The size of the noise component depends on the number of active noise traders in the market.

In view of these facts, some economists argue that certain policy interventions can help to discourage the entry of noise traders in the foreign exchange market and to reduce excess exchange rate volatility. The most popular regulatory policies proposed to "throw sand in the wheels of super-efficient financial vehicles" are to increase entry costs or to levy a Tobin tax type of transaction cost on foreign exchange trading. Proponents of the 'Tobin tax' argues that such a tax may help to reduce market volatility by discouraging short-term destabilizing speculation. In practice, however, it is difficult for authorities to identify noise traders. Policy makers must therefore increase the common entry cost faced by both noise traders and informed traders (traders who base their position on fundamentals) or impose higher transaction taxes on all traders. Consequently, both types of traders will be affected. As a result, the two proposed market stabilization policies might not be as effective as expected.

Since James Tobin proposed the Tobin tax in 1974, the debate about the Tobin tax in foreign exchange market has concentrated on its feasibility and the "distorting" effects it might have as a tax. Several recent theoretical papers question whether an increase in an SST (Securities Transaction Tax) will reduce the volatility of securities prices by discouraging destabilizing investors. For example, see Edwards (1993), Schwert and Seguin (1993), Heaton and Lo (1995), Davidson (1997, 1998), Kupiec (1996), and Song and Zhang (2005). Few papers, however, analyze the effect of a Tobin tax or entry cost on exchange rate volatility in the foreign exchange market. Dooley (1996) argues that short-term speculation may also be stabilizing, so a Tobin tax may not be effective in reducing the exchange rate volatility.

Existing empirical evidence about the relationship between stock transaction costs and equity market volatility mostly suggests that higher transaction costs foster rather than mitigate equity price volatility. For example, Umlauf (1993) studies the effect of transactions taxes on Swedish equity returns in the 1980s and shows that market volatility does not decrease in response to the introduction of a round-trip transaction tax in 1984 or its increase in 1986. Hu

(1998) analyzes 14 tax changes that occurred in Hong Kong, Japan, Korea and Taiwan during 1975-94 and finds that, on average, an increase in tax rate has no impact on market volatility. Findings in Ronen and Weaver (2001) and Bessenbinder (2003) both suggest a positive linkage between transaction costs and equity price volatility. More recently, using panel data, Hau (2006) finds that higher transaction costs in general, and security transaction costs in particular, should be considered as volatility increasing rather than decreasing.[1] Meanwhile, there are only few studies that document the effect of a foreign exchange transaction tax on the volatility of exchange rates. Among them, Aliber, Chowdhry and Yan (2002) estimate the effective transaction costs in the foreign exchange market for the period 1977 to 1999 using foreign currency futures data and find some evidence that a Tobin tax on foreign exchange transactions may increase volatility. Thus, it seems that the empirical evidence is inconsistent with the conventional wisdom that increases in transaction cost can reduce financial price volatility.

Therefore, to understand the effect of entry costs or the Tobin tax on exchange rate volatility, it is important to explore their effects in a model where the trading activity of both noise traders and informed traders is affected. Hence, in this paper, we will investigate this question in a model where both types of traders' entry decisions are endogenized. Jeanne and Rose (2002) consider the endogenous entry of noise traders in a general equilibrium framework. Nevertheless, in their model, informed traders bear no entry cost and always enter the foreign exchange market, while noise traders must pay an entry cost to trade in the market. In this paper, we generalize their model by making entry costly for all traders. Therefore, traders are identical except that informed traders are able to form rational expectations on risks and returns, while noise traders have noisy expectations. We first consider the case where there is no Tobin tax. Then, we investigate the effect of the Tobin tax on the foreign exchange market when both types of traders are subject to entry cost.

Our findings are as follows. When the entry decisions of noise traders and informed traders are both endogenous, there will be three possible equilibria. The noise trading will impose a risk externality on informed traders and affect their entry and exit decisions, while at the same time informed traders' behavior will also affect noise traders' entry benefits. Therefore, this interaction between different types of traders will generate multiple equilibria. In two of the

[1] Hau (2006) also provides a concise summary of the empirical literature on the relationship between transaction costs and financial price volatility.

equilibria, regulatory policies are completely ineffective in reducing exchange rate volatility. In the third equilibrium, increasing entry costs or levying a Tobin tax may even have an adverse effect and lead to an increase in exchange rate volatility. This is because, when the entry of both types of traders is endogenous, in equilibrium, exchange rate volatility increases with the relative noise component (the relative number of noise traders to informed traders). An increase of entry costs or the Tobin tax may not necessarily lead to a decrease in the relative noise component, and thus may be ineffective in volatility reduction.

In the first equilibrium, noise traders and informed traders will enter the market in pairs. Therefore, in the presence of higher entry costs and transaction taxes, noise traders and informed traders will exit in pairs as well. In this equilibrium, the noise component will always be unity. For the second equilibrium, the entry cost is high enough such that noise traders do not enter at all. Hence, the relative noise component is zero. Therefore, the regulatory polices are completely ineffective.

In our model, the deviation of informed traders' expected excess return from unconditional expectations depends on the relative presence of noise traders in the market. Intuitively, informed traders need to take the relative noise component, μ, into consideration when forming rational expectations, while noise traders do not. As a result, the relative noise component affects the gross benefit of entry differently for informed traders than for noise traders. Since this effect works through expectations of traders, we call this the "asymmetric expectation effect". When the relative noise component is 1, it turns out that informed traders' expectations and noise traders' expectations will equally deviate from the unconditional expectations. Hence, the asymmetric expectation effect disappears. The gross benefit of entry of both types of traders is equally affected by μ, which implies an equilibrium with equal numbers of noise traders and informed traders. For the second equilibrium, when the entry cost is high enough, the presence of the asymmetric expectation effect implies that the entry benefit of informed traders will be higher than that of noise traders. This leads to an equilibrium where all noise trader exit the market. In these two equilibria, increasing the entry cost or transaction tax will not change the relative noise component and thus market volatility.

In the third equilibrium, the entry benefits of the two types of traders are equal, but $\mu \neq 1$. This equilibrium is more interesting since the regulatory policy has adverse effects (increasing μ). Intuitively, in this equilibrium, although increases in entry costs or transaction taxes will discourage the entry of both types of traders, the exit of traders will have a relatively positive externality on the gross benefits of noise traders. This is because the exit of informed traders increases the relative noise component, which in turn increases the entry benefits for all traders. Furthermore, because of the asymmetric expectation effect, noise traders will be relatively better-off compared with informed traders. Therefore, increasing entry costs will hurt informed traders more than noise traders. As a result, the relative noise component and the exchange rate volatility will increase.

Our paper is most closely related to Jeanne and Rose (2002). Jeanne and Rose (2002) show that the endogenous entry of noise traders can lead to multiple equilibria in the foreign exchange market and that monetary policy can be used to lower exchange rate volatility without altering macroeconomics fundamentals. Hau (1998) also investigates the free entry of noise traders and finds that temporal noise may result in higher exchange rate volatility and multiple equilibria. In contrast, our paper focuses on the effects of two types of market regulatory policies on the foreign exchange volatility.

Regrading our discussion of the Tobin tax, our study contributes to the literature on the effects of the Tobin tax on foreign exchange rate volatility. Since Tobin (1978) suggested imposing a tax on all foreign exchange transactions, some similar proposals have also often been made by eminent economists (Frankel, 1996, Stiglitz 1989, Summers and Summers, 1989). They claim that a transactions tax will reduce excess noise trading and volatility of exchange rates. The intuitive rationale behind this, first articulated by Keynes in 1936, is that a transaction tax would hurt the speculator disproportionately more because speculators tend to trade much more frequently.

In this paper, we show that the transaction cost may hurt informed traders disproportionately more when both informed traders and noise traders face endogenous entry decisions. This implies that the Tobin tax can increase exchange rate volatility in our model. This result is contrary to the claim made by Tobin and the proponents of the Tobin tax. Moreover, although our model focuses on the foreign exchange market, the insights of our findings should also apply to the more general financial markets. That is, an increase in transaction costs may affect the entry of informed traders as well as the entry of noise traders. Therefore, an increase in the entry cost or transaction taxes may not necessarily lead to a decrease in the noise component. Hence, if the financial market volatility depends on the relative noise component, then an increase in entry costs or transaction taxes may not imply a reduction in the market volatility.[2] Therefore, our findings that a Tobin tax may increase volatility are consistent with

[2]For a more detailed discussion of this issue, please see Section 5.

the evidence presented in the empirical literature on the relationship between transaction costs and equity market volatility and those given by Aliber, Chowdhry and Yan (2002).

Nevertheless, it should be noted that we did not model differences in trading frequency of informed traders and noise traders in this paper. In the model, all traders trade once in each period. A Tobin tax, as discussed above, is often considered as discouraging "hot" short-term capital flows as speculators trade more frequently. However, in our model, we focus on the impact of the Tobin tax on endogenous entry instead of its effect on transaction costs due to different trading frequencies.[3]

This paper is organized as follows. In Section 2, we present a model of the foreign exchange market with endogenous entry of both informed traders and noise traders. Section 3 characterizes the equilibrium and the determination of the exchange rate. We then proceed to investigate the effect of increasing entry costs on the foreign exchange market in Section 4. In Section 5, we analyze the effect of the Tobin tax on the foreign exchange market. Section 6 concludes the paper.

2 Model

We generalize the model of Jeanne and Rose (2002) by making entry costly for all traders. We assume there exists a common entry cost faced by both informed and noise traders. Both types of traders will thus decide whether or not to enter the foreign exchange market at date $t-1$.

2.1 Macroeconomic Fundamentals

We begin with a conventional monetary model of the exchange rate with flexible prices. Using the quantity theory of money and the assumption that the output is decreasing in the interest

[3]To check the proposition that speculators trade more frequently and fundamentalists trade less frequently, Claessens, Dooley and Wagner (1995) measure holding periods of different kinds of capital flow. Empirical evidence reported in their paper suggests that holding periods of direct investors and international holders of "long-term capital" (as defined in balance of payments statistics) are in fact not longer, more persistent over time or more predictable as compared to holding periods of investors in short-term capital. Dooley (1996) also argues that direct investors are likely to be the first to exit a country or a currency when market sentiment turns against the country for good or bad reasons.

rate, we have a Cagan type money demand function, which links the log of the money stock (m_t) deflated by the log of the price level (p_t) to the interest rate (i_t):

$$m_t - p_t = -\alpha i_t. \tag{2.1}$$

We also have the foreign money demand function as below:

$$m_t^* - p_t^* = -\alpha i_t^*. \tag{2.2}$$

Prices are assumed to be perfectly flexible and purchasing power parity is satisfied. Then

$$s_t = p_t - p_t^*. \tag{2.3}$$

Therefore, the exchange rate is determined by

$$s_t = (m_t - m_t^*) + \alpha(i_t - i_t^*). \tag{2.4}$$

As in Jeanne and Rose (2002), we focus on the domestic market and assume that the home bonds is a risky asset. So we assume that the foreign country is in a steady state with constant money supply, price level and interest rates. For simplicity, we set $m^* = p^* = 0$. We also assume that the domestic money supply, m_t, follows a stochastic i.i.d. normal process centered on $\bar{m}$.

2.2 The Home Bond Market and Traders

The interest rate i_t is determined on the bond market where traders choose their portfolios between home and foreign bonds to maximize their utility derived from real returns (or equivalently in terms of foreign currency, as the foreign price level is constant). Since the return on the foreign bond is fixed at i^*, the foreign bond is a risk-free asset while the home bond is a risky asset. Traders are risk averse, so they require a risk premium to hold home bonds.

Moreover, as in Jeanne and Rose (2002), we assume that the supply of home bonds to international investors is fixed at a constant level, $\bar{B}$. This assumption is made for the sake of analytical convenience.

Foreign exchange traders are modeled as overlapping generations of investors who live for two periods and allocate their portfolios between the home and foreign one-period nominal bonds in the first period of their lives. To trade in the home bond market, traders have to enter the foreign exchange market. Furthermore, traders have the same endowments and tastes, but differ in their ability to trade in the home bond market. Some of them are able to form accurate expectations on risk and returns, while others have noisy expectations about future returns. The former are referred to as "informed trader" while the latter are "noise traders". Hereafter, the informed trader is denoted by the superscript I and the noise trader is denoted by the superscript N.

Both noise traders and informed traders have to pay a entry cost to trade on the foreign exchange market. In particular, the entry cost paid by noise traders and informed traders is the same. That is, informed traders and noise traders are identical except that they have different abilities in forming expectations on risks and returns.

In the foreign exchange market, at each period, a generation of foreign exchange traders is born. The traders are indexed by $i = 1, \cdots, N$. We assume that in each generation of traders, N_I of them are informed traders, and $N - N_I$ are noise traders.[4] The timing of the model is illustrated in Figure 1.

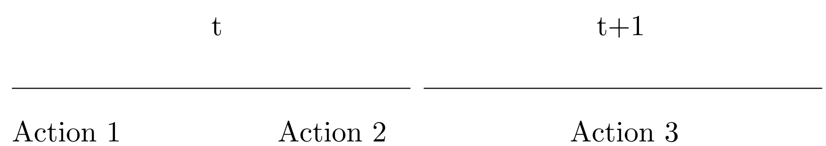

Figure 1: Timing of Model

Action 1: In time t foreign exchange trader i is born. Based on his expectations about time t shocks and the return on home bonds, he decides if he should enter the home bond market.

Action 2: Time t shocks and the return on home bond are realized. Trader i decides on the number of home bonds $B_t(i)$ to purchase based on his expectation about the future exchange rate, S_{t+1}.

Action 3: The time $t+1$ exchange rate S_{t+1} is revealed. The return on trader i' investments in terms of foreign currency are realized. He gets the return, consumes, and dies.

Let φ_t^i denote the dummy variable characterizing the market-entry condition of trader i born in period t. If $\varphi_t^i = 0$, trader i will not enter the home bond market and if $\varphi_t^i = 1$, he

[4]We assume that N and N_I are sufficiently large so that we can guarantee that when the size of the market increases, there will be enough traders.

will enter. At the beginning of period t, trader i will enter the market as long as the expected utility of entering the market is higher than that of not entering:

$$E_{t-1}^{i}(U_t^i \mid \varphi_t^i = 1) \geq E_{t-1}^{i}(U_t^i \mid \varphi_t^i = 0). \tag{2.5}$$

A trader who has entered the home bond market maximizes an exponential utility function:

$$\max_{B_t(i)} E_t^i(-\exp(-aW_{t+1}^i)), \tag{2.6}$$

subject to

$$W_{t+1}^i = W(1+i^*) + B_t^i \left(\frac{1+i_t}{S_{t+1}} - \frac{1+i^*}{S_t} \right) - c_i, \tag{2.7}$$

where W is the initial wealth of every trader in terms of the foreign currency. B_t^i denotes the amount of one-period home currency bonds held by trader i from time t to time $t+1$, a is the coefficient of absolute risk aversion coefficient, the cost c_i reflects the costs associated with entering the foreign bond market for trader i. Trader i's end-of-life wealth is equal to the trader's initial endowment multiplied by the yield on foreign bonds plus, if trader i enters the foreign bond market, the excess return on home bonds minus a fixed cost that must be borne in order to enter the home bond market.

The entry costs may include taxes, information costs and other costs associated with investments in the home bond market. We will investigate the case when the entry costs are considered as common entry costs for both types of traders, such as the fund and reserve requirements to trade in the foreign exchange market as well as taxes.

In the noise trader literature, it is usually assumed that noise traders need to pay a positive entry cost, while informed traders have zero entry cost, as informed traders are assumed to know more about the economic conditions.[5] This assumption works fine for the purpose of analyzing the impact of noise traders on exchange rate volatility, but it is inappropriate if we want to investigate the implication of regulatory policies on foreign exchange markets. When only noise traders pay the entry cost, if the entry cost increases, the noise component in the market will decrease as will the exchange rate volatility. Thus, increasing the entry cost seems to be an effective policy for reducing excess exchange rate volatility caused by non-fundamental shocks.

Nevertheless, in reality, the policy makers cannot distinguish between noise traders and informed traders, so they can only increase the common entry cost faced by both types of traders

[5] See, for instance, Jeanne and Rose (2002) and Hau (1998).

if they want to use this type of policy to reduce excess exchange rate volatility. Consequently, increasing the common cost will affect the entry of both noise traders and informed traders. Thus, it might not reduce the noise component in the market. The fact that policy makers can only affect the common entry cost and thus there might exist a non-monotonic relationship between the common cost and exchange rate volatility is important for our model.

Therefore, we assume that[6]

$$c_i = c \qquad \forall i \in [0,1], \tag{2.8}$$

where c is the level of common entry cost controlled by policy makers.

We may rewrite trader i's problem (if he decides to enter) as follows:

$$\max_{B_t^i} E_t^i \left\{ -\exp\left\{ -a[(1+i^*)W + B_t^i \rho_{t+1} - c_i] \right\} \right\}, \tag{2.9}$$

where the excess return on home bonds between t and $t+1$ is given by:[7]

$$\rho_{t+1} = \left(\frac{1+i_t}{S_{t+1}} - \frac{1+i^*}{S_t} \right) \approx i_t - (s_{t+1} - s_t) - i^*. \tag{2.10}$$

If the excess return is normally distributed,[8] maximizing Equation 2.9 is equivalent to maximizing the mean-variance objective function:

$$E_t^i(W_{t+1}^i) - \frac{a}{2} Var_t^i(W_{t+1}^i) \tag{2.11}$$

Therefore, the optimal demand of trader i for home bonds is given by:

$$B_t^i = \frac{E_t^i(\rho_{t+1})}{aVar_t^i(\rho_{t+1})} \tag{2.12}$$

We now discuss the information structure of traders. Specifically, we make the following assumptions about the subjective distribution over ρ_{t+1}. Informed traders can predict ρ_{t+1} correctly; while noise traders cannot predict future excess returns correctly. That is, for informed traders:

$$E_t^I[\rho_{t+1}] = E_t[\rho_{t+1}] \tag{2.13}$$

[6] From our numerical analysis, we find that qualitative results of the model will not change even if we assume that noise traders pay a higher entry cost than informed traders pay. As long as both types of traders have to pay some positive entry costs to enter the market, our main result holds.

[7] Here, we use an approximation: if ξ is small enough, $\log(1+\xi) = \xi$. $\frac{1+i_t}{S_{t+1}} - \frac{1+i^*}{S_t} = (\frac{1+i_t}{S_{t+1}} - 1) - (\frac{1+i^*}{S_t} - 1) \approx \log(\frac{1+i_t}{S_{t+1}}) - \log(\frac{1+i^*}{S_t})$. Using the approximation again, we can derive Equation 2.10.

[8] This is true in equilibrium, as we show below.

$$Var_t^I[\rho_{t+1}] = Var_t[\rho_{t+1}] \tag{2.14}$$

For noise traders, following Jeanne and Rose (2002), we assume that:

$$E_t^N[\rho_{t+1}] = \bar{\rho} + v_t \tag{2.15}$$

$$Var_t^N[\rho_{t+1}] = Var_t[\rho_{t+1}] \tag{2.16}$$

$$Var(v_t) = \lambda Var(s_t) \qquad \text{where } \lambda \in (0, +\infty), \tag{2.17}$$

where $\bar{\rho}$ is the unconditional mean of the excess return and v_t is assumed to be $i.i.d$ and normally distributed with zero mean. λ can be considered as a parameter characterizing the relative magnitude of noise traders' erroneous beliefs in exchange rate volatility.

From Equations 2.13 and 2.15, it can be seen that, compared with informed traders' expectations, noise traders' expectations of ρ_{t+1} based on time t information is biased from the true conditional expectations. Nevertheless, noise traders can correctly forecast the conditional variance of the exchange rate. From Equation 2.17, another assumption is made that the unconditional variance of v_t is proportional to the unconditional variance of the exchange rate itself. This assumption helps to tie down the scale of the volatility of noise traders' erroneous beliefs.[9]

3 Equilibrium

Equilibrium for this economy is a collection of four sequences $\{s_t, \rho_t, \varphi_t^i, B_t^i\}$, such that at each period t, φ_t^i satisfies the entry condition (2.5), B_t^i is the solution of the optimal bond holding problem (2.12), and the market clearing condition for home bonds,

$$\bar{B} = \sum_{i=1,\cdots,N} \varphi_t^i B_t^i, \tag{3.1}$$

holds. Since the equilibrium involves heterogenous individual traders' decision rules in a stochastic environment, the equilibrium is difficult to determine. However, if the shocks are i.i.d, it turns out that the set of equilibrium individual decision rules takes a simple stable form.

[9]The logic behind this assumption is that the bias in noise traders' expectations must be related to the volatility of the exchange rate itself, otherwise noise traders might expect the future exchange rate to be volatile even under a fixed exchange rate regime.

As in Jeanne and Rose (2002), we solve the model with a "guess and verify" approach. We first postulate the properties of the equilibrium and then we check if they are satisfied. We conjecture that:

(i) the log of the exchange rate, s_t, is $i.i.d$ around an average level $\bar{s}$;

(ii) a constant number of informed traders, x, and a constant number of noise traders, n, enter the home bond market in every period, where $x \leq N_I$ and $n \leq N - N_I$.

Using these properties, we can solve for the equilibrium of the model. We will first derive entry conditions of the two types of traders. Then, in the next two sections, we analyze the model with the entry cost and the Tobin Tax, respectively.

3.1 Exchange Rate

The home bond market clearing condition implies:[10]

$$\begin{aligned} \bar{B} &= x\frac{E_t(\rho_{t+1})}{aVar_t(\rho_{t+1})} + n\frac{(\bar{\rho}+v_t)}{aVar_t(\rho_{t+1})} \\ &= \frac{xE_t(\rho_{t+1}) + n(\bar{\rho}+v_t)}{aVar_t(\rho_{t+1})}, \end{aligned} \tag{3.2}$$

where x and n are the number of informed traders and noise traders entering the home bond market, respectively. Note that $0 < x \leq N_I$ and $0 < n \leq N - N_I$. Therefore, from Equation 3.2, we derive the expression for the time t conditional expectation for the $t+1$ risk premium:

$$E_t(\rho_{t+1}) = \frac{a\bar{B}Var(s) - n(\bar{\rho}+v_t)}{x}. \tag{3.3}$$

This equation shows that the conditional expected return of informed traders is affected by n, the number of active noise traders on the market. Also, taking the unconditional expectation of 3.2, we could get the average risk premium:

$$\bar{\rho} = a\frac{\bar{B}}{x+n}Var(s). \tag{3.4}$$

This implies that the average risk premium is proportional to the coefficient of absolute risk aversion, the average trading volume, and exchange rate volatility.

[10] Using the equilibrium property of s_t, $Var_t(\rho_{t+1}) = Var_t(s_{t+1}) = Var(s)$

Finally, we derive the equilibrium exchange rate using the property of s_t in equilibrium:[11]

$$s_t - \bar{s} = \frac{1}{1+\alpha}\left(m_t - \bar{m} - \alpha\frac{n}{x}v_t\right) \tag{3.5}$$

Since m_t and v_t are both *i.i.d.* normal, this expression confirms that the log of the exchange rate is *i.i.d.* normal in equilibrium.

From Equation 3.5, we have the first proposition that shows how exchange rate volatility is determined in this model.

Proposition 1 *In equilibrium, exchange rate volatility increases with the fundamentals, $Var(m)$, and the relative noise component in the market, μ.*

$$Var(s) = \frac{Var(m)}{(1+\alpha)^2 - \lambda\alpha^2\mu^2}, \tag{3.6}$$

where $\mu = \frac{n}{x}$ and satisfies

$$0 \leq \mu < \frac{1+\alpha}{\alpha\sqrt{\lambda}}. \tag{3.7}$$

The proof is trivial. Given fundamental shocks, exchange rate volatility is fully determined by the composition of traders. For convenience, we denote the composition of traders as μ. The equilibrium μ is endogenously determined by entry decisions of both informed traders and noise traders. To have a positive exchange rate variance in equilibrium, μ must be bounded. This condition implies that there must exist a positive number of informed traders in the market. In Jeanne and Rose (2002), all informed traders are in the market, so exchange rate volatility is only determined by the number of noise traders who endogenously enter the market. However, when we consider the entry of both noise traders and informed traders, exchange rate volatility will be determined by the relative number of noise traders to informed traders. This also implies that policies that discourage the entry of traders are not necessarily effective in reducing the noise component.

[11]Equation 3.5 can be derived as follows. First, from Equation 2.4, we have $s_t - \bar{s} = m_t - \bar{m} + \alpha(i_t - \bar{i})$. Second, from Equations 3.2 and 3.3, we have $E_t(\rho_{t+1}) = \bar{\rho} - \frac{n}{x}v_t$. Third, taking expectations of Equation 2.10, we have $i_t = E_t(\rho_{t+1}) + [E_t(s_{t+1}) - s_t] + i^*$ and $\bar{i} = \bar{\rho} + i^*$. The i.i.d property of s_t in equilibrium implies that $i_t - \bar{i} = E_t(\rho_{t+1}) - \bar{\rho} + \bar{s} - s_t = -\frac{n}{x}v_t + \bar{s} - s_t$. Substituting this into Equation 2.4 gives us 3.5.

3.2 Entry Condition

For any trader, as long as its entry cost is less than the gross benefit of entry, the trader will enter the foreign exchange market; that is,

$$\text{if } c_i \leq GB(\bar{\rho}, Var(s)), \;\; \varphi_t^i = 1 \qquad\qquad i \in (1, N). \tag{3.8}$$

where GB is the gross benefit of entry. We denote GB_N and GB_I as the gross benefit of entry for noise traders and informed traders, respectively. From the Technical Appendix,[12] we have

$$GB_N = \frac{\bar{\rho}^2}{2aVar(s)(1+\lambda)} + \frac{1}{2a}\log(1+\lambda); \tag{3.9}$$

$$GB_I = \frac{\bar{\rho}^2}{2aVar(s)(1+\mu^2\lambda)} + \frac{1}{2a}\log(1+\mu^2\lambda). \tag{3.10}$$

Equations 3.9 and 3.10 show that the effect of the noise component, μ, on the gross benefit of entry for noise traders and informed traders is asymmetric. μ will affect GB_I and GB_N through its impact on $Var(s)$ and $\bar{\rho}$. When μ increases, it will increase the variance in the exchange rate and raise the risk premium. This is the 'create their own space' effect discussed by De Long et al. (1990). Meanwhile, if the increase in μ is due to the presence of more noise traders, it also increases the total number of traders demanding the risky assets, which reduces the amount of risk born by all traders since the market is deeper and lowers the risk premium. This is the 'market depth' effect in the finance literature. Besides this, an increase in μ also has an extra negative impact on GB_I, as shown by 3.10.

In short, this is because μ will affect informed traders' expectations of excess return but not noise traders' expectations. In particular, from (3.3) and (3.4) we have

$$E_t^I\rho_{t+1} = E_t\rho_{t+1} = \bar{\rho} - \mu v_t, \qquad E_t^N\rho_{t+1} = \bar{\rho} + v_t \tag{3.11}$$

That is, the noise trader's expected excess return is always given by $\bar{\rho}+v_t$, but informed traders need to take the presence of noise traders in the foreign exchange market into consideration when forming their rational expectations. When μ is higher, informed traders' expectations have to deviate more from $\bar{\rho}$ to make certain that, on average, the expected return of all market participants is equal to $\bar{\rho}$. That is, $xE_t^I\rho_{t+1} + nE_t^N\rho_{t+1} = x(\bar{\rho} - \mu v_t) + n(\bar{\rho} - v_t) = (x+n)\bar{\rho}$. Hence, an increase in μ will have a negative impact on informed traders since this leads to

[12] The derivation for noise traders' entry benefits is exactly the same as in Jeanne and Rose's (2002) appendix.

a higher deviation of their expected return from $\bar{\rho}$, and lowers the expected return on their investments. This will in turn reduce the gross benefit of entry for informed traders.

Therefore, the asymmetric effect of μ on informed traders' and noise traders' expectations of excess return implies that the relative noise component affects the gross benefit of entry differently for informed traders than for noise traders. Since this effect works through expectations of traders, we call this the "asymmetric expectation effect" on informed traders, which represents the extra effect of μ on GB_I.

To further analyze properties of the gross benefit of entry for traders, we substitute Equations 3.4 and 3.6 into the above expressions, and then we have

$$GB_I = \frac{1}{2a(1+\mu^2\lambda)} \frac{a^2\bar{B}^2}{(x+n)^2} \frac{Var(m)}{[(1+\alpha)^2 - \lambda\alpha^2\mu^2]} + \frac{1}{2a}\log(1+\mu^2\lambda); \tag{3.12}$$

$$GB_N = \frac{1}{2a(1+\lambda)} \frac{a^2\bar{B}^2}{(x+n)^2} \frac{Var(m)}{[(1+\alpha)^2 - \lambda\alpha^2\mu^2]} + \frac{1}{2a}\log(1+\lambda). \tag{3.13}$$

From Equations 3.12 and 3.13, we make the following proposition,

Proposition 2 *If N and N_I are sufficiently large, then changes in $\bar{B}$ and $Var(m)$ will have no effect on the gross benefit of entry for traders and the composition of traders.*

The proof is simple. In equilibrium, the number of each type of trader who enter the market is constant and given by (x^*, n^*), so the noise component is then given by $\mu^* = \frac{n^*}{x^*}$, which satisfies $0 \leq \mu^* < \frac{1+\alpha}{\alpha\sqrt{\lambda}}$. If $\bar{B}$ changes to $\kappa\bar{B}$, where $\kappa > 0$, then $(\kappa x^*, \kappa n^*)$ will be the new equilibrium solution which gives the same noise component μ^*. Therefore, the gross benefit of entry for noise traders or informed trader is not affected. Therefore, we can argue that changes in the size of the market will only have impact on the number of active traders, but not on the composition of traders and in turn on the exchange rate volatility.

The proof for the effect of changes of $Var(m)$ is similar. When $Var(m)$ increases by a factor of κ, then $(\sqrt{\kappa}x^*, \sqrt{\kappa}n^*)$ will be the new equilibrium solution which implies the same noise component μ^*. From this proposition, we can also identify an interesting property of the equilibrium, which is different from Jeanne and Rose (2002). In their model, there is a non-monotonic relation between fundamentals and exchange rate volatility. If the $Var(m)$ is below a threshold, there is a unique equilibrium. If the $Var(m)$ is above a much higher threshold, the equilibrium is again unique. Between these two thresholds, there exists a "zone of multiplicity". However, in our model, since changes in $Var(m)$ have no effect on μ, we can find a linear relationship between $Var(s)$ and $Var(m)$.

In the following sections, we investigate the effect of increases in the entry cost and the Tobin Tax on the equilibrium of the model. We first consider the case where there is no Tobin tax. Then, we investigate the effect of the Tobin tax on the foreign exchange market when both types of traders are subject to entry costs.

4 Entry Cost

In this case, each trader i faces a common entry cost, c. Following Jeanne and Rose (2002), we assume that the entry cost is not too small, that is, $c > \frac{1}{2a}\log(1+\lambda)$. From the gross benefit function of entry for noise traders and informed traders and from the fact that μ must be bounded in equilibrium, we have the following lemma.

Lemma 1 *For any entry cost $c > \frac{1}{2a}\log(1+\lambda)$, if N and N_I are sufficiently large, then in equilibrium, we must have*

$$GB_N \leq c; \tag{4.1}$$

$$GB_I = c. \tag{4.2}$$

The proof is straightforward. From the function of GB_N, we can see that the gross benefit of entry for noise traders is composed of two parts. The first part depends on μ, while the second part is constant. We only need to focus on the first part when we analyze the entry of noise traders. Meanwhile, to have a positive variance of exchange rates in the market, the noise component must satisfy $0 \leq \mu < \frac{1+\alpha}{\alpha\sqrt{\lambda}}$. From 3.13, since μ is bounded and the first component of GB_N is a decreasing function of n, we know that when n increases, eventually $GB_N \leq c$ and noise traders will stop entering. Therefore, in equilibrium, we must have $GB_N \leq c$.[13]

Regarding the entry of informed traders, the fact that μ is non-negative and bounded implies that the number of informed traders in the market must be positive. That is, $x > 0$. Therefore, $GB_I < c$ is not a possible equilibrium. Meanwhile, as shown by 3.12, the first part of GB_I is a decreasing function of x when μ is bounded, and the second part of GB_I is an increasing function of μ. Given the functional form of μ, when x increases, μ and the second

[13]Note that we assume that the number of each type of trader is sufficiently large, so that $GB_N > c$ is not possible.

part of GB_I will decrease. Therefore, if N_I is sufficiently large, eventually GB_I will equal c in equilibrium. In other words, in equilibrium, we must have $GB_I = c$.[14]

Given the above lemma, we can analyze the properties of possible equilibria. We will first look at the equilibria where $GB_I = GB_N = c$. For this case, we have the following propositions that give characteristics of the two equilibria.

Proposition 3 *For any entry cost, $c > \frac{1}{2a}\log(1+\lambda)$, there always exists an equilibrium with $\mu = 1$. In equilibrium,*

(i) the number of active traders, n^*, x^* ($n^* = x^* = \bar{x}$), will be determined by:

$$GB_N = GB_I = \frac{1}{2a(1+\lambda)}\frac{a^2\bar{B}^2}{(2\bar{x})^2}\frac{Var(m)}{[(1+\alpha)^2 - \lambda\alpha^2]} + \frac{1}{2a}\log(1+\lambda) = c; \tag{4.3}$$

(ii) n^* and x^* increase with fundamental shocks, $Var(m)$, and the financial market size, $\bar{B}$, and decrease with entry cost, c.

(iii) increasing the entry cost c has no effect on exchange rate volatility.

The proof is trivial. In equilibrium, if $\mu = 1$, we have $GB_N = GB_I$. This implies that GB_I and GB_N are both strictly decreasing functions of the number of entrants. Note that the first term of GB_N (GB_I) is always positive. Therefore, as long as $c > \frac{1}{2a}\log(1+\lambda)$, there will be a unique (n^*, x^*) determined by c. A larger market size and higher fundamental volatility can increase the gross benefit of entry for traders. Hence, given the level of entry cost, more traders will enter the foreign exchange market when $Var(m)$ and $\bar{B}$ increase.

In this equilibrium, an increase in the entry costs will reduce the number of entrants. Nevertheless, noise traders and informed traders will leave the market in pairs, so the relative noise component will remain unchanged. Therefore, increasing the entry costs will be ineffective in reducing exchange rate volatility.

This is an interesting result. From Equation 3.11, the deviation of informed traders' expectations of the excess return from unconditional expectations depends on the relative presence of noise traders in the market. That is, informed traders will take the relative noise component, μ, into consideration when forming expectations of the excess return. When $\mu = 1$, informed traders' expectations and noise traders' expectations will be equally deviate from $\bar{\rho}$.[15] This

[14] We will analyze the case when N and N_I are not sufficiently large later.

[15] That is, if noise traders are optimistic (pessimistic), informed traders will be equally pessimistic (optimistic) to make sure that the market average expectation is $\bar{\rho}$.

implies that the 'asymmetric expectation effect' of μ on informed traders disappears. The presence of μ will not create any extra asymmetric impact on the gross benefit of informed traders relative to that of noise traders. GB_I and GB_N will then be equally affected by μ through its impact on $Var(s)$ and $\bar{\rho}$, which implies that $\mu = 1$ is an equilibrium. Furthermore, in this equilibrium, the gross entry benefit of both types of traders only depends on the market depth, $x + n$, the total number of market participants. Therefore, as long as noise traders and informed traders can enter and leave the market in pairs, increasing the entry cost will only change the market depth, but not the composition of noise traders and informed traders, and $\mu = 1$ can still be an equilibrium.

The $\mu = 1$ equilibrium is not the only equilibrium when $GB_N = GB_N = c$. There exists another equilibrium given in the following Proposition.

Proposition 4 *For any entry cost $\frac{1}{2a}\log(1+\lambda) < c < \bar{c}$, there also exists an equilibrium with $\mu \neq 1$, which is characterized by $GB_N = GB_I = c$. In equilibrium,*

(i) the noise component, μ, is determined by the following condition,

$$GB(\mu) = c \quad \text{if} \quad \frac{1}{2a}\log(1+\lambda) < c < \bar{c}, \tag{4.4}$$

where $GB(\mu)$ denotes a function of the noise component, μ, $GB(\mu) = \frac{1}{2a(\mu^2-1)\lambda}[(1+\mu^2\lambda)\log(1+\mu^2\lambda) - (1+\lambda)\log(1+\lambda)]$ and $\bar{c} = GB(\frac{1+\alpha}{\alpha\sqrt{\lambda}})$.

(ii) the noise component μ increases with the entry cost. That is, $\frac{\partial\mu}{\partial c} > 0$, which implies that increasing entry cost will lead to high exchange rate volatility.

The proof is given in the Appendix. In this equilibrium, $\mu \neq 1$. Hence, the 'asymmetric expectation effect' implies that μ will affect the gross benefit of entry for noise traders and informed traders differently. Therefore, it is impossible to have $x + n$ decrease in response to the increase in c, while μ remains unchanged. In particular, when the entry cost increases, μ increases and exchange rate volatility increases. This implies that increasing the entry cost has an adverse effect on market volatility.

Why is this the case? A higher entry cost will discourage the entry of both noise traders and informed traders. In this model, the exit of an informed trader and the exit of a noise trader both reduce market depth (which implies an increase in the risk borne by all traders and the risk premium), but they have an opposite impact on $\mu = \frac{n}{x}$. The former increases μ, while the latter decreases μ. Given the market depth, increases (decreases) in μ, the relative noise component, will lead to increases (decreases) of risk premium and the gross benefit of entry. Moreoever, since $\mu \neq 1$ in this equilibrium, the 'asymmetric expectation effect' implies that changes in μ will affect the gross benefit of entry differently for informed traders than for noise traders.

Therefore, on the one hand, when an informed trader exits the market, μ increases, which implies that noise traders will gain more than informed traders. On the other hand, when a noise trader leaves the market, μ decreases. And this will benefit informed traders more than noise traders. Nevertheless, the exit of an informed trader increases μ, while the exit of a noise trader implies a decrease in μ, so the former will lead to a larger increase in entry benefits for all traders than the latter. Thus, the effects due to the exit of informed traders will dominate those due to the exit of noise traders. In other words, the exit of traders has a relatively positive externality on the gross benefit of noise traders. In some sense, we can say that increasing the entry cost will hurt informed traders more than it will hurt noise traders. As a result, in a new equilibrium where $GB_I = GB_N = c$, the relative noise component will increase, which leads to higher exchange rate volatility.

This result shows that, in this equilibrium, increasing the entry costs is ineffective as a regulatory policy in reducing excess exchange rate volatility. It is worth noting that this equilibrium depends critically on the range of c. This is because the equilibrium, μ, is bounded, so when the entry cost c is above the threshold $\bar{c} = GB(\frac{1+\alpha}{\alpha\sqrt{\lambda}})$, this equilibrium will not exist any more.[16]

When $GB_N < c$ and $GB_I = c$, we can establish another equilibrium.

Proposition 5 *If c is sufficiently large, there also exists a third equilibrium where no noise traders enter the market, while the number of informed traders, x, is determined by the following condition*

$$GB_I(x) = \frac{1}{2a}\frac{a^2\bar{B}^2}{x^2}\frac{Var(m)}{(1+\alpha)^2} = c. \tag{4.5}$$

The condition for the existence of this type of equilibrium is that the entry cost satisfies the

[16] If there is an initial equilibrium with $\mu > 1$, then when the entry cost moves beyond this threshold, the equilibrium will switch to the equilibrium when noise traders and informed enter the market in pairs (the $\mu = 1$ equilibrium). Therefore, in this special case, increasing the entry costs may reduce exchange rate volatility. But after that, increasing the entry costs will have no effect on exchange rate volatility.

following condition,

$$c = GB_I > GB_N \Rightarrow c > \frac{1}{2a}\frac{1+\lambda}{\lambda}\log(1+\lambda). \tag{4.6}$$

This proposition says that when the entry cost is high enough, there will be an equilibrium with zero noise traders. In this equilibrium, increasing the entry cost is also ineffective in reducing exchange rate volatility (note that when $\mu = 0$, exchange rate volatility is still positive).[17] Why do no noise traders enter the market in this equilibrium? When the entry cost increases, the gross benefit of entry to all traders has to increase, which can be achieved through a decrease of market depth or an increase in the relative noise component, μ. Nevertheless, as shown in Equation (3.7), the relative noise component, μ, is bounded. Hence, if the entry cost is high enough, the increase of GB_I and GB_N can not rely on the increase of the noise component. In other words, in this case, the increase in the entry benefit should mainly be achieved through decreases in market depth. Furthermore, since μ is bounded, the decrease in market depth mostly comes from decreases in n.

As discussed above, the exit of noise traders will reduce μ, which has a positive 'asymmetric expectation effect' for informed traders and increases their gross benefit of entry. Therefore, when c is high enough, the presence of the asymmetric expectation effect implies that $GB_I = c > GB_N$. Hence, in this equilibrium, there are zero noise traders in the market.

Note that in all three equilibria listed above, the average risk premium in the market will increase with the entry cost. This is because the higher the average risk premium is, the larger the gross benefit of entry for traders will be. Therefore, it is easy for traders to survive the market with a higher average risk premium as it can compensate for traders' higher participation costs.

The equilibria described in Propositions 3-5 are based on an implicit assumption that the number of each type of trader is sufficiently large. Therefore, there is no equilibrium where

[17] We check the stability properties of the equilibria discussed in Proposition 3, 4, and 5. For the $\mu = 1$ and $\mu \neq 1$ equilibria, due to the complexity of the algebra, we can not get an analytical condition for the stability of the equilibria, so we have to resort to numerical solutions. Tables 1 and 2 in the Technical Appendix give numerical results for the stability properties of these two equilibria. Our numerical exercises show that λ, the parameter which measures the relative magnitude of noise traders' erroneous beliefs to exchange rate volatility, is the key factor to determine the stability of the equilibria. In particular, we find that, when λ is small, $\mu = 1$ is a stable equilibrium, but $\mu \neq 1$ is not a stable equilibrium. But when λ is large, the stability property of the equilibria reverses. That is, the $\mu \neq 1$ equilibrium is stable. In the Technical Appendix, we also show that the $\mu = 0$ equilibrium is always stable.

all informed traders or noise traders enter in the foreign exchange market. However, if we relax this assumption, there could exist a new equilibrium that is similar to the one described in Jeanne and Rose (2002).[18] In this equilibrium, all informed traders are in the market; therefore, increasing the entry costs only affects on noise traders. Although the assumption that N and N_I are limited is not very realistic, this equilibrium makes it easy to compare our result with that of Jeanne and Rose (2002). We describe this equilibrium in the following proposition.

Proposition 6 *If N and N_I are not sufficiently large, there also exists another equilibrium where all informed traders enter the market, while the number of noise traders is determined by the following condition*

$$GB_N(n) = \frac{1}{2a(1+\lambda)} \frac{a^2\bar{B}^2}{(x+n)^2} \frac{Var(m)}{[(1+\alpha)^2 - \lambda\alpha^2\mu^2]} + \frac{1}{2a}\log(1+\lambda) = c, \tag{4.7}$$

where $x = N_I$ and $u = \frac{n}{N_I}$. The condition for the existence of this equilibrium is that, in equilibrium, the structural parameters satisfy the following condition

$$GB_I = \frac{1}{2a(1+\mu^2\lambda)} \frac{a^2\bar{B}^2}{(N_I+n)^2} \frac{Var(m)}{[(1+\alpha)^2 - \lambda\alpha^2\mu^2]} + \frac{1}{2a}\log(1+\mu^2\lambda) > GB_N = c, \tag{4.8}$$

where $\mu = \frac{n}{N_I}$ and n is determined by (4.7).

The properties of this equilibrium will be exactly the same as those in Jeanne and Rose (2002). In this equilibrium, the noise component, μ, may not be unique, and there might exist multiple equilibria. In this case, the effect of entry costs on exchange rate volatility is a little

[18]If the number of noise traders is not large enough, there also exist other equilibria where $GB_N > c$ and $GB_I \geq c$. That is, all noise traders enter the market. Two possible equilibria exist:

i) $GB_N > c$, so $n = N - N_I$; and $GB_I = c$, so x is determined by $GB_I(x) = c$, where $\mu = \frac{N-N_I}{x}$. Note that the condition for this equilibrium to exist is that, in equilibrium, we must have $GB_N > GB_I$.

ii) $GB_N > c$, so $n = N - N_I$; and $GB_I > c$, so $x = N_I$.

Nevertheless, these two equilibria are not very interesting since the fact that all noise traders enter implies that increasing c cannot reduce the number of noise traders in the market. Furthermore, in case i), increases in c will only reduce x and increase μ, the noise component; while in case ii), increases in c will have no impact on the noise component, μ. Finally, their existence is subject to the fact that $N - N_I$ is limited, which is not very practical. Hence, we will not discuss these cases in the text.

Table 1: The Effect of Entry Cost Changes on the Foreign Exchange Market

(New Equilibria compared to Jeanne and Rose, 2002[a])

	c=0.25		c=0.3	
Equilibrium	Prop. 3 Eq.	Prop. 4 Eq.	Prop. 3 Eq.	Prop. 4 Eq.
μ	1	0.45	1	1.02
n	127.6	43.2	99.3	106.3
x	127.6	95.5	99.3	104.3
$x+n$	255.2	138.7	198.6	210.6
GB_I	0.25	0.25	0.3	0.3
GB_N	0.25	0.25	0.3	0.3
$Var(s)$	1	0.295	1	1.12
$\bar{\rho}$	1.57	0.85	2.01	2.14

[a]: We do not include the equilibrium described in Proposition 5, since μ always equals zero in this case.

bit complicated. In general, if the entry costs are small and the number of informed traders is not large enough, then this type of equilibrium will exist. However, firstly, increasing c may push the economy away from this equilibrium towards the equilibria described in Proposition 3-5. Hence, when c increases, the noise component, μ, may either increase or decrease, as will exchange rate volatility. However, once the economy stays in one of the equilibria given in Propositions 3-5, increasing c will not be effective in reducing foreign exchange volatility anymore. Second, even if the economy stays in the Jeanne and Rose's equilibria, due to the existence of multiple equilibria, it is still difficult to judge the effect of an increase of the entry cost on exchange rate volatility. Thus, even in this case, the regulatory policy may still be ineffective. Of course, if we focus on one of the Jeanne and Rose's equilibria, we can have a decrease of $Var(s)$ when c increases.

To show the effects of the entry cost on the foreign exchange market, we also report some numerical results in Tables 1 and 2. Following Jeanne and Rose (2002), we set $\alpha = 1$, $a = 4$, $\lambda = 3$, $N_I = N - N_I = 200$, $\bar{B} = 100$, $Var(m) = 1$. Therefore, we have $0 < \mu < \frac{1+\alpha}{\alpha\sqrt{\lambda}} = 1.15$ and the entry cost $c > \frac{1}{2a}\log(1+\lambda) = 0.17$. The results in Tables 1-2 confirm our findings in Propositions 3-5.

Table 2: The Effect of Entry Cost Changes on the Foreign Exchange Market

(Jeanne and Rose's Equilibrium with $\lambda = 1.5$)

	c=0.22	c=0.25	c=0.28	c=0.3
μ	0.427	0.227	0.102	0.039
n	42.66	22.69	10.16	3.87
x	100	100	100	100
$x+n$	142.66	122.69	110.16	103.87
GB_I	0.24	0.32	0.41	0.46
GB_N	0.22	0.25	0.28	0.3
$Var(s)$	0.2683	0.2549	0.2510	0.2501
$\bar{\rho}$	0.75	0.83	0.91	0.96

5 Tobin Tax

Now we investigate the case when both noise traders and informed traders pay a transaction tax, i.e., a Tobin tax, in addition to the common entry cost.

The model is almost the same as before. There are two kinds of traders: informed traders and noise traders. They decide whether or not to enter the domestic bond market based on $t-1$ information. It should be noted that since there are only two types of traders in the model, all trades occur across the two types and any transaction costs would therefore fall on both types of traders equally. The analysis of a transaction tax therefore is equivalent to the study of entry costs as discussed in the previous section.

For any trader i, if he chooses not to enter the home bond market, his expected utility at date $t-1$ is given by

$$E^i_{t-1}(U^i_t \mid \varphi^i_t = 0) = -\exp[-a(1+i^*)W]. \tag{5.1}$$

If he chooses to enter, his expected utility is:

$$E^i_{t-1}(U^i_t \mid \varphi^i_t = 0) = -\exp[-a(1+i^*)W]E_{t-1}\left\{\max_{B^i_t}\left[\exp(-aB^i_t\rho_{t+1} + ac_i - \frac{1}{2}\tau B^{i2}_t)\right]\right\}, \tag{5.2}$$

where τ is the Tobin tax on the bond transaction.[19] Therefore, the optimal bond holding for

[19]If the Tobin tax is applied to the level of bond holdings, then the tax rate τ can be positive or negative

trader i is given by

$$B_t^i = \frac{E_t^i(\rho_{t+1})}{\tau + aVar_t^i(\rho_{t+1})}. \tag{5.3}$$

Thus, in the presence of a Tobin tax, the volume of bond transactions will be reduced. Given the information structure of traders in Section 2, we have

$$B_t^I = \frac{E_t(\rho_{t+1})}{\tau + aVar_t(\rho_{t+1})}, \qquad B_t^N = \frac{\bar{\rho} + v_t}{\tau + aVar_t(\rho_{t+1})}. \tag{5.4}$$

The home bond market clearing condition implies that :

$$\begin{aligned} \bar{B} &= x\frac{E_t(\rho_{t+1})}{aVar_t(\rho_{t+1})} + n\frac{(\bar{\rho} + v_t)}{aVar_t(\rho_{t+1})} \\ &= \frac{xE_t(\rho_{t+1}) + n(\bar{\rho} + v_t)}{\tau + aVar_t(\rho_{t+1})}. \end{aligned} \tag{5.5}$$

Now we can derive the average risk premium and the exchange rate in equilibrium, respectively:

$$\bar{\rho} = \frac{\bar{B}\tau + a\bar{B}Var(s)}{x+n}, \tag{5.6}$$

$$s_t - \bar{s} = \frac{1}{1+\alpha}\left(m_t - \bar{m} - \alpha\frac{n}{x}v_t\right). \tag{5.7}$$

These equations also yield:

$$E_t(\rho_{t+1}) = \frac{\bar{B}\tau + a\bar{B}Var(s) - n(\bar{\rho} + v_t)}{x}, \tag{5.8}$$

$$Var(s) = \frac{Var(m)}{(1+\alpha)^2 - \lambda\alpha^2\mu^2}. \tag{5.9}$$

In this case, the entry conditions are still given by Equation 3.8. As shown in the Technical Appendix, the gross benefit of entry conditions for noise traders and informed traders are given as below:

$$GB_I = \frac{\bar{\rho}^2}{2[\tau + aVar(s) + a\mu^2\lambda Var(s)]} + \frac{1}{2a}\log[1 + \mu^2\frac{a\lambda Var(s)}{\tau + aVar(s)}]; \tag{5.10}$$

$$GB_N = \frac{\bar{\rho}^2}{2[\tau + aVar(s) + a\lambda Var(s)]} + \frac{1}{2a}\log[1 + \frac{a\lambda Var(s)}{\tau + aVar(s)}]. \tag{5.11}$$

The following proposition establishes the properties of the exchange rate in the equilibrium with a Tobin tax.

depending on the net position of bond holdings, which will make our analysis much more complicated. So for simplicity and tractability, we assume that the Tobin tax is applied to the square of bond holdings. Qualitatively, this assumption should lead to similar results as the one with a linear tax rate.

Proposition 7 *In the presence of a Tobin tax, given fundamental shocks, the equilibrium exchange rate and its variance are still completely determined by the noise component, μ. If there is no entry cost, then the Tobin tax has no effect on exchange rate volatility. The Tobin tax only increases the average risk premium on the foreign exchange market, and this effect depends on the market size, $\bar{B}$.*

When there is a Tobin tax, the determination of exchange rate volatility does not change. The Tobin tax has no direct effect on exchange rate volatility. It can only affect exchange rate volatility through the composition of traders. From Equations 5.10 and 5.11, the gross benefit of entry for both types of traders is always positive. That is, $GB(\bar{\rho}, Var(s)) \geq 0$, $\forall i \in (1, N)$. Therefore, without an entry cost, no matter how large the Tobin tax is, all traders will always enter the foreign exchange market. In other words, if $c = 0$, the noise component, μ, will be exogenously determined by the ex-ante distribution of traders, i.e., $\mu = \frac{N - N_I}{N_I}$. This implies that the volatility of the exchange rate will be independent of τ. From Equation 5.6, however, we know that the average risk premium increases with τ. This is because, in the presence of a Tobin tax, traders will require higher returns to compensate for the extra transaction cost of trading risky bonds.

The effect of the Tobin tax on the gross benefits is complicated. It has a direct negative effect on the gross benefits for both types of traders since τ shows up in the denominator of GB_I and GB_N. However, the Tobin tax also can affect the gross benefits of entry through its effect on the average risk premium, $\bar{\rho}$, and the noise component, μ. Nevertheless, from the expression of $\bar{\rho}$ and $Var(s)$, given τ, Lemma 1 still holds in this case. That is, as in Section 4, if N and N_I are sufficiently large, in equilibrium, we must have $GB_I = c$ and $GB_N \leq c$. Thus, in the following propositions, we can outline the properties of the equilibria when there are both Tobin taxes and entry costs.

Proposition 8 *In the presence of both entry costs and Tobin taxes, there still exists an equilibrium with $\mu = 1$, where the Tobin tax has no effect on exchange rate volatility. Meanwhile, there exists another equilibrium, where $GB_I = GB_N = c$, but $\mu \neq 1$.*

The proof of this proposition is straightforward. In any case, as long as noise traders and informed traders enter the market in pairs, then their gross benefit of entry will be the same. Imposing a transaction tax on foreign exchange trading will only force noise traders and informed traders to leave the market in pairs, but it will have no impact on μ and exchange rate volatility.

Meanwhile, as in Proposition 4, in the presence of both entry costs and Tobin taxes, there still exists an equilibrium where $GB_I = GB_N = c$, but $\mu \neq 1$. Nevertheless, in this equilibrium, it is difficult to analyze the effect of the Tobin tax on exchange rate volatility, as the impact of the Tobin tax on the gross benefit of entry is quite complicated. We resort to numerical analysis and report our findings in Table 3. We set $\tau = 0.02$ so that the transaction cost is about 0.25% of the value of bond trading. Table 3 shows that the effect of the Tobin tax on the entry of traders is small. The Tobin tax slightly reduces the number of informed traders while it increases the number of noise traders. As a result, the noise component and exchange rate volatility increase. Therefore, in this experiment, levying a Tobin tax on foreign exchange trading will have an adverse effect on the exchange rate volatility.

The intuition behind this result is similar to that for Proposition 4. Note that the impact of the noise component, μ, on GB_I and GB_N is asymmetric. An increase in τ will reduce the gross benefit of entry for both noise traders and informed traders. Nevertheless, the exit of informed traders (an increase in μ) will partially offset the negative impact of an increase in τ. Therefore, it is possible that when τ increases, the noise component and exchange rate volatility will increase. As shown above, the Tobin tax will only affect the entry of traders when they need to pay a cost to enter the market. Therefore, in a sense, we can also say that this effect is due to the interaction between the Tobin tax and the entry cost.

Finally, similar to the equilibrium described in Proposition 5, in this case, for a large entry cost, c, we can still have an equilibrium where $GB_I = c$ and $GB_N < c$. Given τ, this implies that $n = 0$ and x is determined by $GB_I(x, 0) = c$. In other words, the noise component is always zero in this equilibrium. Therefore, increasing the Tobin tax has no impact on exchange rate volatility in this equilibrium.

In this paper, we focus on the foreign exchange market and discuss the impact of increases in entry costs and transaction taxes on exchange rate volatility. Nevertheless, the main insights of our paper should apply to more general financial markets. Specifically, for any financial market where there are two types of traders, informed traders and noise traders (whose information sets and utility functions are the same as those in our model), the main results of this paper should hold as long as the excess return of the risky asset over the risk-free asset (ρ_t in the paper) is assumed to be *i.i.d.* This is because, given the assumption that the excess return is i.i.d, we can derive the optimal demand for the risky assets. Then, as shown in the Technical

Table 3: The Effect of Tobin Tax on the Foreign Exchange Market

(Equilibrium with $c = 0.25$ and $\lambda = 3$)

	$\tau = 0$		$\tau = 0.02$	
Equilibrium	$\mu = 1$	$\mu \neq 1$	$\mu = 1$	$\mu \neq 1$
μ	1	0.45	1	0.469
n	127.6	43.18	127.8	44.82
x	127.6	95.54	127.8	95.51
$x + n$	255.2	138.72	255.6	140.33
GB_I	0.25	0.25	0.25	0.25
GB_N	0.25	0.25	0.25	0.25
$Var(s)$	1	0.2952	1	0.2994
$\bar{\rho}$	1.567	0.851	1.573	0.8678

Appendix, we can derive the gross benefits of entry for informed and noise traders accordingly (Equations (3.9) and (3.10)). These two equations are essential for the derivation of other major results of the model, such as Propositions 3, 4 and 5.

The insights emphasized in this paper are that when we consider the endogenous entry of both kinds of traders, an increase in the entry cost or transaction taxes may affect the entry of informed traders as well as the entry of noise traders. Therefore, an increase in the entry cost or transaction taxes may not necessarily lead to a decrease in the noise component. Hence, if the financial market volatility depends on the relative noise component, then an increase in c or τ may not imply a reduction in the market volatility. In a more general financial market, the functional form of the volatility of financial asset prices will be different, but the logic and the intuition should still apply.

6 Conclusion

This paper examines the effects of two types of market regulatory policies in reducing excess exchange rate volatility caused by noise trading in the foreign exchange market. In practice, policy makers cannot distinguish informed traders from noise traders. Hence, in this paper, we consider the entry decisions of both types of traders. We generalize the model of Jeanne and Rose (2002) by making entry costly for all traders. We find that if the number of traders is large enough, there exist three equilibria in this model. In these equilibria, increasing entry costs or imposing Tobin taxes may be ineffective in reducing exchange rate volatility, or even have an adverse effect and increase exchange rate volatility.

This is because the volatility of exchange rates is determined by the composition of traders in the foreign exchange market. In two equilibria, the relative ratio of noise traders (the noise component) is constant. In the third equilibrium, the impact of the noise component on the gross benefits of entry for noise traders and informed traders is asymmetric. When there is an increase in the entry cost in the foreign exchange market, this may not change the relative ratio of traders, or it may affect informed traders disproportionately more, which will increase the relative ratio of noise traders and exchange rate volatility.

Regarding the Tobin tax, our finding is surprising. When there is endogenous entry of both types of traders, imposing a Tobin tax may not affect exchange rate volatility, or may even increase exchange rate volatility. This finding is the opposite of what proponents of the Tobin tax have argued. The intuition is that the interaction between the Tobin tax and the entry cost may lead to an increase in the relative noise component and exchange rate volatility. Therefore, our findings suggest that future research on the Tobin tax should focus more on its interaction with entry costs in the foreign exchange market.

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Appendix

A Proof of Proposition 4

The gross benefit of entry for noise traders and informed traders is given as below

$$GB_N = \frac{\bar{\rho}^2}{2aVar(s)(1+\lambda)} + \frac{1}{2a}\log(1+\lambda) \tag{A.1}$$

$$GB_I = \frac{\bar{\rho}^2}{2aVar(s)(1+\mu^2\lambda)} + \frac{1}{2a}\log(1+\mu^2\lambda) \tag{A.2}$$

From Equations A.1 and A.2, we can derive the following relation between GB_N and GB_I,

$$(1+\lambda)(GB_N - \frac{1}{2a}\log(1+\lambda)) = (1+\mu^2\lambda)(GB_I - \frac{1}{2a}\log(1+\mu^2\lambda)) \tag{A.3}$$

In this equilibrium (note that $\mu \neq 1$), $GB_I = GB_N = c$, thus, we can show that $GB_I = GB_N$ is a function of μ. So after rearranging Equation A.3, we have

$$GB(\mu) = \frac{1}{2a(\mu^2-1)\lambda}[(1+\mu^2\lambda)\log(1+\mu^2\lambda) - (1+\lambda)\log(1+\lambda)] = c \tag{A.4}$$

For any $0 \leq \mu < \frac{1+\alpha}{\alpha\sqrt(\lambda)}$, we have $GB(\mu) > 0$. To analyze the effect of c on μ, we need to check the sign of $\frac{\partial c}{\partial \mu}$. Its sign is the same as that of $\frac{\partial c}{\partial \mu^2}$.

$$\frac{\partial c}{\partial \mu^2} = \frac{(1+\lambda)[\ln(1+\lambda) - \ln(1+\mu^2\lambda)] + \lambda(\mu^2-1)}{2a(\mu^2-1)^2\lambda} \tag{A.5}$$

For convenience, we denote

$$f(\mu^2) = (1+\lambda)[\ln(1+\lambda) - \ln(1+\mu^2\lambda)] + \lambda(\mu^2-1) \tag{A.6}$$

Since $2a(\mu^2-1)^2\lambda$ is always positive, the sign of $\frac{\partial \mu}{\partial c}$ is determined by the sign of $f(\mu^2)$. To check the sign of $f(\mu^2)$, we look at the partial derivative of $f(\mu^2)$ with respect to μ^2 as below

$$f'(\mu^2) = \frac{\lambda^2(\mu^2-1)}{1+\mu^2\lambda} \tag{A.7}$$

Note that $\mu \geq 0$, so for any $\mu > 1$, $f'(\mu^2) > 0$, and for any $\mu < 1$, $f'(\mu^2) < 0$. Meanwhile, since when $\mu^2 = 1$, $f(\mu^2) = 0$, for any μ, we have

$$f(\mu^2) > 0 \tag{A.8}$$

This implies

$$\frac{\partial \mu}{\partial c} > 0 \tag{A.9}$$

QED.

Technical Appendix

(Not to Published)

A The Gross Benefit of Entry

This technical appendix derives the gross benefit of entry for both noise traders and informed traders. We first state the following lemma that has been proved by Jeanne and Rose (2002).

Lemma 1 *If $\tilde{x}$ is normally distributed with mean zero and variance σ^2, and μ_0, μ_1, and μ_2 are scalars (with $\mu_2 > -\frac{1}{2\sigma^2}$), then*

$$E\left\{\exp[-(\mu_0 + \mu_1\tilde{x} + \mu_2\tilde{x}^2)]\right\} = \frac{1}{\sqrt{1+2\mu_2\sigma^2}}\exp(-\mu_0 + \frac{\mu_1^2\sigma^2}{2(1+2\mu_2\sigma^2)}) \tag{A.1}$$

We apply this lemma in the following subsection to derive the gross benefit of entry for both inform traders and noise traders.

A.1 Informed trader

In the presence of a Tobin tax, if trader i enters the foreign exchange market, his/her expected utility is given by

$$EU = \Lambda E_{t-1}^{i}\left\{\max_{B_t(i)}\left[\exp\left(-aB_t(i)\rho_{t+1} + ac_i + a\frac{\tau}{2}B_t^2(i)\right)\right]\right\} \tag{A.2}$$

where $\Lambda = -\exp[-a(1+i^*)W]$ is the expected utility of trader i is he/she does not enter the market. The optimal bond holding for informed traders is given by

$$B_t(i) = \frac{E_t(\rho_{t+1})}{\tau + aVar_t(\rho_{t+1})} \tag{A.3}$$

Substituting Equation A.3 into Equation A.2, and rearranging it, we have

$$EU = \Lambda E\left\{\exp[(A-\mu v_t)^2 B - \frac{a}{\tau + aVar_t(s)}(A-\mu v_t)\eta_{t+1} + ac_i]\right\} \tag{A.4}$$

where

$$A = \frac{\bar{B}(\tau + aVar(s)) - n\bar{\rho}}{x} \tag{A.5}$$

$$B = \frac{-a(\tau + 2aVar(s))}{2(\tau + aVar(s))^2} \tag{A.6}$$

$$\eta_{t+1} = \rho_{t+1} - \bar{\rho} - v_t \tag{A.7}$$

and $\eta_{t+1} \sim N(0, Var(s))$.

Since the two stochastic variable v_t and η_{t+1} are independent, as in Jeanne and Rose (2002), we can apply the Lemma to each variables successively. Applying the Lemma to η_{t+1}, we have

$$\tilde{x} = \eta_{t+1} \tag{A.8}$$

$$\mu_0 = -B(A - \mu v_t)^2 - ac_i \tag{A.9}$$

$$\mu_1 = (A - \mu v_t)\frac{a}{\tau + aVar(s)} \tag{A.10}$$

$$\mu_2 = 0 \tag{A.11}$$

Therefore, we have

$$EU = \Lambda E_{t-1}\{\exp[A^2 + \mu^2 v_t^2 - 2\mu A v_t)(-\frac{a}{2[\tau + aVar(s)]}) + ac_i]\} \tag{A.12}$$

Now we apply the Lemma to v_t. In the second step, we get

$$\tilde{x} = v_t \tag{A.13}$$

$$\mu_0 = \frac{aA^2}{2[\tau + aVar(s)]} - ac_i \tag{A.14}$$

$$\mu_1 = -\mu\frac{Aa}{[\tau + aVar(s)]} \tag{A.15}$$

$$\mu_2 = \mu^2\frac{a}{2[\tau + aVar(s)]} \tag{A.16}$$

Note that

$$Var(v) = \lambda Var(s) \tag{A.17}$$

Therefore, we have

$$EU = \Lambda\sqrt{\frac{\tau + aVar(s)}{\tau + aVar(s) + \mu^2 aVar(v)}}\exp\left\{\frac{-aA^2}{2[\tau + aVar(s) + \mu^2 aVar(v)]} + ac_i\right\} \tag{A.18}$$

Now we can derive the entry condition for informed trader i. He/she will enter if and only if $EU \geq \Lambda$, so the entry condition becomes:

$$\sqrt{\frac{\tau + aVar(s)}{\tau + aVar(s) + \mu^2 aVar(v)}}\exp\left\{\frac{-aA^2}{2[\tau + aVar(s) + \mu^2 aVar(v)]} + ac_i\right\} \leq 1 \tag{A.19}$$

Since $A = \bar{\rho}$ from Equation 5.6, taking the log of this expression and using the identity $Var(v) = \lambda Var(s)$, we could get

$$GB_I \geq c_i \tag{A.20}$$

where GB_I is defined as the gross benefit of entry for informed traders and is given by

$$GB_I = \frac{\bar{\rho}^2}{2[\tau + aVar(s) + \mu^2\lambda Var(s)a]} + \frac{1}{2}\log[1 + \mu^2\frac{a\lambda Var(s)}{\tau + aVar(s)}] \tag{A.21}$$

If we set $\tau = 0$, then we can get the gross benefit of entry for informed traders without the Tobin tax.

A.2 Noise traders

In the presence of the Tobin tax, the optimal bond holding for noise traders is given by

$$B_t(i) = \frac{\bar{\rho} + v_t}{\tau + aVar_t(\rho_{t+1})} \tag{A.22}$$

Substituting Equation A.22 into Equation A.2, and rearranging it, we have

$$EU = \Lambda E_{t-1}^{i}\{exp[B(\bar{\rho} + v_t)^2 - \frac{a(\bar{\rho} + v_t)}{\tau + aVar(s)}\eta_{t+1} + ac_i]\} \tag{A.23}$$

where η_{t+1} and v_t are as defined in Section A.1. Applying the Lemma to η_{t+1}, we get

$$\tilde{x} = \eta_{t+1} \tag{A.24}$$

$$\mu_0 = -B(\bar{\rho} + v_t)^2 - ac_i \tag{A.25}$$

$$\mu_1 = \frac{a(\bar{\rho} + v_t)}{\tau + aVar(s)} \tag{A.26}$$

$$\mu_2 = 0 \tag{A.27}$$

Therefore, we have

$$EU = \Lambda E_{t-1}\left\{exp\left[\frac{-a}{2[\tau + aVar(s)]}(\bar{\rho}^2 + v_t^2 + 2\bar{\rho}v_t) + ac_i\right]\right\} \tag{A.28}$$

Applying the Lemma to v_t, in this step we get

$$\tilde{x} = v_t \tag{A.29}$$

$$\mu_0 = \frac{a}{2[\tau + aVar(s)]}\bar{\rho}^2 - ac_i \tag{A.30}$$

$$\mu_1 = -\frac{a}{2[\tau + aVar(s)]}\bar{\rho} \tag{A.31}$$

$$\mu_2 = \frac{a}{2[\tau + aVar(s)]} \tag{A.32}$$

Therefore, we have

$$EU = \Lambda\sqrt{\frac{\tau + aVar(s)}{\tau + aVar(s) + aVar(v)}}\exp\left\{\frac{-a\bar{\rho}^2}{2[\tau + aVar(s) + aVar(v)]} + ac_i\right\} \tag{A.33}$$

Similarly, using $EU \geq \Lambda$, we can drive the gross benefit of entry for noise traders

$$GB_N = \frac{\bar{\rho}^2}{2[\tau + aVar(s) + \lambda Var(s)a]} + \frac{1}{2}\log[1 + \frac{a\lambda Var(s)}{\tau + aVar(s)}] \tag{A.34}$$

If we set $\tau = 0$, then we can get the gross benefit of entry for noise traders without Tobin tax.

B Stability of Equilibria

We discuss the stability property of the $\mu = 1$ and $\mu \neq 1$ equilibria first. In our model, given n, $GB^I(n, x) = c$ determines x, while given x, $GB^N(n, x) = c$ determines n. So the equilibrium (n,x) can be solved by the following 2 equations:

$$GB^I(n, x) - c = 0 \tag{B.35}$$

$$GB^N(n, x) - c = 0 \tag{B.36}$$

To check the stability of equilibria, first, we evaluate the Jacobian matrix A at the equilibrium point($n = n^*$, $x = x^*$),

$$A = \begin{pmatrix} \frac{\partial GB^I}{\partial n} & \frac{\partial GB^I}{\partial x} \\ \frac{\partial GB^N}{\partial n} & \frac{\partial GB^N}{\partial x} \end{pmatrix} \tag{B.37}$$

Second, we calculate the eigenvalues of matrix A. There should be two eigenvalues in our case. Eigenvalues are generally complex numbers. If the real parts of all eigenvalues are negative, then the equilibrium is stable. If at least one eigenvalues has a positive real part, then the equilibrium is unstable.

Obviously, due to the algebra complexity, we can not get any analytical condition for the stability of equilibria, so we have to resort to numerical solution. Following Jeanne and Rose (2002), we set $\alpha = 1$, $a = 4$, $N_I = N - N_I = 200$, $\bar{B} = 100$, $Var(m) = 1$ and $c = 0.25$. The tables below give the numerical results of the stability property of the $\mu = 1$ and $\mu \neq 1$ equilibria, for the case of $\lambda = 1$ and $\lambda = 3$, respectively. Our numerical exercises show that λ, the parameter which measures the relative magnitude of noise traders' erroneous beliefs to exchange rate volatility, is the key factor to determine the stability of the equilibria. In particular, we find that, when λ is small, $\mu = 1$ is a stable equilibrium, but $\mu \neq 1$ is not a stable equilibrium. But when λ is large, the stability property of the equilibria reverses. That is, the $\mu \neq 1$ equilibrium is stable.

This is because when λ is small, the size of noise traders' erroneous beliefs will be small, so the difference between noise traders and inform traders' information sets will be reduced. This implies that the risk externality inflicted by noise traders on informed traders will also be smaller. Thus, neither type of traders will deviate from the $\mu = 1$ equilibrium. When λ increases, the incentive for deviation increases since the difference between noise traders and informed traders also increases. Hence, the $\mu = 1$ equilibrium will not be a stable equilibrium.

Regarding the $\mu = 0$ equilibrium described in Proposition 5, in equilibrium, we must have $GB_N < c$. Hence, for small deviation of n and x, this inequality will still hold. This implies that we have $n = 0$ and $\mu = 0$. Therefore, to find the stability property of this equilibrium, we only need to check if $\frac{\partial GB^I}{\partial x}|_{x^*>0,n^*=0} < 0$, where x^* is given by Equation 4.5. It is easy to show that $\frac{\partial GB^I}{\partial x}|_{x^*>0,n^*=0}$ is always negative. So the $\mu = 0$ equilibrium is always stable.

Table 1: The stability of equilibria ($\lambda = 1$)

	$\mu = 1$	$\mu \neq 1$
n	71.4	78.37
x	71.4	124.07
μ	1	1.5832
A	$\begin{pmatrix} -0.0013 & -0.0033 \\ -0.0008 & -0.0038 \end{pmatrix}$	$\begin{pmatrix} 0.0020 & -0.0055 \\ 0.0028 & -0.0086 \end{pmatrix}$
e	$\begin{pmatrix} -0.0005 \\ -0.0046 \end{pmatrix}$	$\begin{pmatrix} 0.0002 \\ -0.0069 \end{pmatrix}$
stability	stable	unstable

Table 2: The stability of equilibria ($\lambda = 3$)

	$\mu = 1$	$\mu \neq 1$
n	127.65	95.5
x	127.65	43.2
μ	1	0.4520
A	$\begin{pmatrix} 0.0036 & -0.0048 \\ 0.0030 & -0.0042 \end{pmatrix}$	$\begin{pmatrix} -0.0023 & -0.0029 \\ -0.0005 & -0.0014 \end{pmatrix}$
e	$\begin{pmatrix} 0.0006 \\ -0.0012 \end{pmatrix}$	$\begin{pmatrix} -0.0031 \\ -0.0006 \end{pmatrix}$
stability	unstable	stable